SOLUTIONS

225 MIDTERM 2, SPRING 2007

TUESDAY, APRIL 3

(1) **(8 points)** Is the following set of three vectors linearly independent?

$$\left\{\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}2\\0\\5\end{pmatrix},\begin{pmatrix}3\\1\\3\end{pmatrix}\right\}$$

$\begin{pmatrix}1&2&3\\0&0&1\\0&5&3\end{pmatrix}$ ↕ switch $\longrightarrow \begin{pmatrix}1&2&3\\0&5&3\\0&0&1\end{pmatrix}$ echelon form

$\Rightarrow$ full rank

Yes

(2) **(12 points)**

(a) Write down the matrix A that describes a counter-clockwise rotation of the plane around the origin around the angle θ.

$$\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$$

Date: April 3, 2007.

(b) From a geometric point of view, what is A^2?

The counter-clockwise rotation around zero by the angle 2θ $\begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$

(c) Is A an orthogonal matrix?

Yes $A^T \cdot A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} =$

$= \begin{pmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \sin^2\theta + \cos^2\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(3) **(18 points)** True or false:

(a) b is in the column space of A if and only if b is a linear combination of the column vectors of A.

true (that is the definition of the column space)

(b) $Ax = b$ has a solution if and only if b is in the column space of A.

true

(c) 0 is always a solution of $Ax = b$.

false: if $b \neq 0$, then $A \cdot 0 + b$

(d) 0 is always in the column space of A.

true: $0 = 0 \cdot \vec{v}_1 + \ldots + 0 \cdot \vec{v}_m$, where the $\vec{v}_i$ are the column vectors of A.

(e) x is a solution of $Ax = b$ if and only if x is in the column space of A.

false

(f) Two vectors always span a plane.

false, they could span a line, e.g. $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 6 \end{pmatrix}$.