

Observation: The Eisenstein series $g_n = 2 \cdot G_n$ satisfy all these conditions.

At $p=2$, the story is not as clean...

$$\pi_q \text{ fibre } (gl_2 \text{ tmf} \rightarrow L_2 gl_2 \text{ tmf}) = 0 \quad \forall q > 3$$

More generally, if $R = L_n R$ and R is Eois ring spectrum, then

$$\pi_q \text{ fibre } (gl_2 R \rightarrow L_n gl_2 R) = 0 \quad \forall q > n+1.$$

The key input is:

$$K(n) \otimes K(q, \mathbb{Z}/p) = 0 \quad \forall q > n.$$

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$L_{K(2)}$ tmf

Let R be an \mathbb{F}_p -algebra, C/R elliptic curve.

C is super-singular if \tilde{C} has strict height 2.

Example: $y^2 + y = x^3 / \mathbb{F}_2$

Classification: Over a field $R = k$, \tilde{C} curves are classified by $j(C) \in k$ if $k = \bar{k}$ is algebraically closed.

Super-singular curves correspond to a finite set of j -values.

$\mathcal{M}_{SS}(R) =$ groupoid of super-singular curves over R

$=$ the locus where $v_1 = 0$

$$v_2 \in \Gamma(\mathcal{M}_{ell} \otimes \mathbb{F}_p, \omega^{\otimes p-2})$$

At $p=0$, there is more than one iso class of ss. elliptic curves, so the limit is a product of history fixed point spectra.

Fact: for $p \geq 5$, $p \nmid \#G$.

What is the elliptic curve?

Serre-Tate deformation theory for abelian varieties :

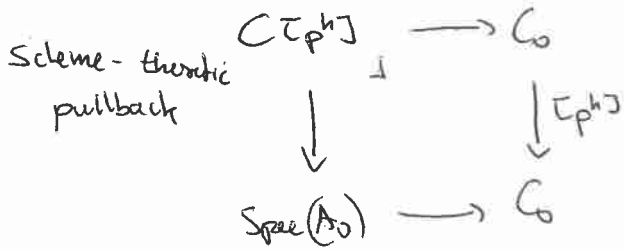
A ring, I ideal nilpotent, $p \in I$,

$$A_0 = A/I$$

$$\eta_{\text{ell}}(A) \xrightarrow{\text{reduction}} \eta_{\text{ell}}(A_0)$$

$$\cong \text{Def}(A, A_0)$$

- objects :
- G/A_0 elliptic curve
 - p -divisible group G/A
 - isomorphism $E: G_0 \xrightarrow{\cong} G_0[p^{\infty}]$



p -divisible group = inductive limit of such group schemes.

$$G_0[p^h]^1 \twoheadrightarrow G_0[p^h] \twoheadrightarrow G_0[p^h]^{\text{et}}$$

formal part

etale part

(= 0 for strict height 2)

This is G_0 is super-singular, then all we need to get a lift on A is a lift of the formal group. Lubin-Tate they do this, and so there is a unique lift.

\Rightarrow "Near the locus of super-singular curves, the morphism

$$\eta_{\text{ell}} \rightarrow \eta_{FG} \text{ is etale?}$$