In the case of $s\mathcal{S}^{Ass}$, $\pi_i^\natural(X,P)$ or $E_i^\natural(X)$ behave like htpy. grps. of a simple space - so it has Postnikov decomp.

$$\begin{tikzcd} P_n X_\bullet \arrow[r] \arrow[d] & P_0 X_\bullet \arrow[d] \\ P_{n-1} X_\bullet \arrow[r] & B(A, n+1) \end{tikzcd}$$

has $\pi_0^\natural$ and $\pi_{n+1}^\natural$

In the algebraic world, $E_* X_\bullet$ is a simpl. algebra in $E_*E$ - comodules. Here we also has Postnikov sections in the simpl. direction and corresponding Eilenberg-MacLane objects $K(A, n+1)$. But

$$E_* B(A, n+1) \neq K(A, n+1)$$

defined using $\pi_*^\natural$

defined using $\pi_*(\pi_*(-,P))$.

The relationship is given by the spiral exact sequence. In particular,

$$P_{n+1}^{alg} E_* B(A, n+1) = K(A, n+1)$$

Prop The map of simpl. mapping spaces

$$\mathrm{Map}/_{P_0}(X, B(A,n+1)) \xrightarrow{E_*} \mathrm{Map}/_{E_* P_0}(E_* X, K(A,n+1))$$

is a weak equivalence.

pf Formal argument reduces to check the case $X = T(\Delta[q]/\partial\Delta[q] \wedge P)$ with $P \in \mathcal{P}$. This case is proved by calc.:

$$s\mathcal{S}^{A_\infty}(T(\Delta[q]/\partial\Delta[q] \wedge P), B(A,n+1))$$

$$= s\mathcal{S}(\Delta[q]/\partial\Delta[q] \wedge P, B(A,n+1))$$

$$= \pi_q^{\natural}(B(A,n+1), P).$$

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