Dec 1 TMF

A map $\begin{tikzcd} \mathrm{Spec}\, R \arrow[d, "J"] \\ \mathcal{M}_{Ell} \end{tikzcd}$ corresponds to an elliptic curve J over R.

Map between elliptic curves

$$\begin{tikzcd} \mathrm{Spec}\, S \arrow[r] \arrow[rr, bend left] \arrow[dr] & \mathrm{Spec}\, R_{12} \arrow[r] \arrow[d] & \mathrm{Spec}\, R_2 \arrow[d, "J_2"] \\ & \mathrm{Spec}\, R_1 \arrow[r, "J_1"'] & \mathcal{M}_{Ell} \end{tikzcd}$$

to give a map $R_{12} \to S$ is the same as $R_1 \xrightarrow{f_1} S$, $R_2 \xrightarrow{f_2} S$, $f_1^* J_1 \overset{t}{\simeq} f_2^* J_2$

$\Updownarrow$

$\mathrm{Spec}\, S \to \mathrm{Spec}\, R_{12}$

$\mathrm{Aff}/\mathcal{M}_{Ell}$: ob $\mathrm{Spec}\, R_1 \to \mathcal{M}_{Ell}$

map
$$\begin{tikzcd} \mathrm{Spec}\, R_{12} \arrow[r] \arrow[d] & \mathrm{Spec}\, R_2 \arrow[d] \\ \mathrm{Spec}\, R_1 \arrow[r] \arrow[u, bend left] & \mathcal{M}_{Ell} \end{tikzcd}$$

This is true for any stack (don't use property about elliptic curves)

CONCRETE WAY TO DESCRIBE THIS CATEGORY

i.e. ob (R, J) J = ell. curve over R

map $(R_2, J_2) \to (R_1, J_1)$
$R_2 \xrightarrow{f} R_1$ + iso $f^* J_2 \overset{t}{\simeq} J_1$

Fact:

- $\mathcal{M}_{Ell} \longrightarrow \mathcal{M}_{FG}$ is flat
- $\mathcal{M}_{Weier} \longrightarrow \mathcal{M}_{FG}$ is not flat

$y^3 = x^2$ is the bad point

But removing it:

- $\mathcal{M}_{Weier} \setminus \{y^3 = x^2\} \longrightarrow \mathcal{M}_{FG}$ is flat

this $\Rightarrow$

$$\begin{tikzcd} \mathrm{Spec} \arrow[r, "flat"] \arrow[dr, "\therefore flat"'] & \mathcal{M}_{Ell} \arrow[d] \\ & \mathcal{M}_{FG} \end{tikzcd}$$

$\rightsquigarrow$ Landweber exact homology theories

Let

$\mathcal{C}^{SS}_{Top}$ the category with

super singular

topological

objects:

A_∞, even, periodic E together with an elliptic curve

$$\begin{tikzcd} \mathrm{Spec}\, \pi_0 E \arrow[r, "J"] \arrow[dr, "G_E"'] & (\mathcal{M}_{Ell})^{\wedge}_{SS} \arrow[d] \\ & \mathcal{M}_{FG} \end{tikzcd}$$

étale

maps:

A_∞: $E_2 \xrightarrow{f} E_1$

+ an iso $(\pi_0 f)^* J_2 \sim J_1$ of elliptic curves.