$$\frac{z^2+zt_0+\cancel{t_0^2}-(t_1^2+t_1t_0+\cancel{t_0^2})}{z-t_1} = z+t_0+t_1 = 0$$

for colinear pts. Taking inverse we see that the group law for this curve is the additive grp. t_0+t_1. //

ex $C: y^2-xy=x^3$ or $s-st=t^3$ or

$$s=\frac{t^3}{1-t} \quad \text{with} \quad s=\frac{1}{y} \text{ and } t=\frac{x}{y}.$$

$H^0(C,\Omega^1)$ = 1-dim'l gen. by

$$\varphi = \frac{dt}{1-t} = \frac{ds}{3t^2}$$

$$C\times C \xrightarrow{\mu} C$$

$$H^0(C,\Omega^1)\oplus H^0(C,\Omega^1) \xleftarrow{\mu^*} H^0(C,\Omega^1)$$

$$\pi_1^*\varphi+\pi_2^*\varphi \longleftarrow\!\shortmid \quad \varphi$$

If we integrate to get the function

$$\ell(t) = \int\frac{dt}{1-t} = -\log(1-t),$$

this gives the equation

$$\ell(t_1 \underset{c}{+} t_2) = \ell(t_1)+\ell(t_2)$$

or, with $e(x) = \ell^{-1}(x)$,

$$t_1 \underset{c}{+} t_2 = e(\ell(t_1) + \ell(t_2))$$

$$= 1 - (1-t_1)(1-t_2) = t_1 + t_2 - t_1 t_2$$

— the multiplicative group. "

A formal group law over a ring R is a power series

$$x \underset{F}{+} y = F(x,y) \in R[[x,y]]$$

that satisfies

(unital) $x \underset{F}{+} 0 = 0 \underset{F}{+} x = x$

(commutative) $x \underset{F}{+} y = y \underset{F}{+} x$

(associative) $(x \underset{F}{+} y) \underset{F}{+} z = x \underset{F}{+} (y \underset{F}{+} z)$.

Let φ be the unique differential form which is invariant w.r.t. to F, i.e.

$$\varphi(x \underset{F}{+} y) = \varphi(x) + \varphi(y),$$

and which starts out as dx at