

Say height of  $G \geq h$ . If  $g'(c)$  is a unit, say height  $G = h$ .

Suppose  $\text{ht } G = h$

$$[p](x) = u \cdot x^{p^h} + \dots$$

$$= u \cdot x^{p^h} \underbrace{(1 + o(x))}_{\text{unit in } \mathbb{R}[[x]]}$$

unit in  $\mathbb{R}[[x]]$

so  $\mathbb{R}[[x]] / [p](x)$  free  $\mathbb{R}$ -mod. rk.  $h$ .

Start with  $G$  over general ring  $R$ .

$$v_0 = p$$

$$G \text{ over } R/(p) : [p](x) = g(x^p) = v_1 x^p + \dots$$

$$v_1 \in R/(p)$$

$$G \text{ over } R/(p, v_1) : [p](x) = v_2 x^{p^2} + \dots$$

⋮

$$v_n \in R/(p, v_1, \dots, v_{n-1}) : [p](x) = v_n x^{p^n} + \dots$$

Rem The ideal gen. by  $v_n \in \mathbb{R}/(p, v_1, \dots, v_{n-1})$  is independent of  $x$ , i.e. an invariant of the formal group.

Pf  $y = \lambda x + \dots$   $v_n \mapsto \lambda^{p^n-1} v_n \dots //$   
 $\lambda \in \mathbb{R}^*$

### Spectra and homology theories

Def A spectrum is a collection of spaces

$$E = \{E_n\}_{n \in \mathbb{Z}}$$

together with homeomorphisms

$$E_n \xrightarrow[\cong]{t_n} \Omega E_{n+1}$$

A map of spectra is

$$E = \{E_n, t_n^E\} \xrightarrow{f} F = \{F_n, t_n^F\}$$

$$\begin{array}{ccc} E_n & \xrightarrow{f_n} & F_n \\ \cong \downarrow t_n^E & & \cong \downarrow t_n^F \\ \Omega E_{n+1} & \xrightarrow{\Omega f_{n+1}} & \Omega F_{n+1} \end{array}$$