

Simplicial objects :

Δ = (skeleton) category of finite ordered sets

Objects : $[n] = \{0 < 1 < \dots < n\}$, $n \geq 0$.

$\Delta \rightarrow \text{Spaces}$

$[n] \mapsto \Delta^n = \text{standard } n\text{-simplex}$

$\downarrow \Theta \quad \downarrow \Theta_n = \text{affine linear}$

$[m] \mapsto \Delta^m$

A simplicial object in cat. \mathcal{C} is a functor

$$\Delta^{op} \rightarrow \mathcal{C}$$

ex X space, $[n] \mapsto \text{Map}(\Delta^n, X) = \text{Sim}_n(X)$ is a simplicial set.

Cosimplicial obj. are functors

$$\Delta \rightarrow \mathcal{C}$$

ex $[n] \mapsto \Delta^n$ cosimpl. space Δ^{\sim} .

If $P.$ is a simpl. A_0 -ring spectrum then $A_0(P., F)$ is a cosimplicial space.

Def If X' is a cosimpl. space, then

$Tot(X') =$ Space of maps Δ^i to X'

$$\subset \coprod_{n \geq 0} \text{Map}(\Delta^n, X^n)$$

An elem. of $Tot(X')$ consists of a pt. $x_0 \in X^0$, a path in X^1 from $d^0 x_0$ to $d^1 x_0$, ---

Cannot necessarily form

$$X' \rightsquigarrow \pi_k X' \text{ cosimpl. ab. grp}$$

— base-point problem.

Suppose we start with $x_0 \in X^0$ and a path in X^1 from $d^0 x_0$ to $d^1 x_0$. Get a map of cosimpl. spaces

$$sk_1 \Delta^1 \longrightarrow X'$$

Since $sk_1 \Delta^1$ is connected, we get