

Jacob

If B is nice as A algebra
(for Artin Stack : smooth)

(10)

but this condition is too strong ~~for application in ltpy theory~~ and for our purposes it is enough to assume flat.

If you want to do lots of geometry, you want it to be Artin Stack. Apart from that maybe flat is enough.

Homotopy theory

Suppose that B is a (naively) commutative & associative ring spectrum.

$$\pi_* E \Rightarrow \pi_*(E \wedge E) \leftarrow$$

$\rightarrow E \wedge E$ is, too

assume flat

also assume that both of these are concentrated in even degrees (get rid of graded commutativity)

Jacobson $\pi_* E \rightrightarrows \pi_* E \wedge E$

(11)

$A \xrightarrow[\pi_2^*]{\pi_1^*} B$

$B \xrightarrow{\Delta^*} A$

swap^{*} : $B \rightarrow B$

$m^* : B \rightarrow B \otimes_{\mathbb{Z}} B$

uses flatness \Rightarrow \downarrow
 $\pi_*(E \wedge E \wedge E)$
 \uparrow
 $\pi_*(E \wedge S^1 \wedge E)$

\leadsto Hopf algebroid.

ring spectrum E

\leadsto Hopf algebroid \leadsto

groupoid object in schemes

Why is this useful?

ASS

to compute some approximation of htpy sps of spheres

Artin stacks (almost)

E_2 -term turns out to be the cohomology of the structure sheaf of the stack. (absolutely clear from defn. but ignoring the grading)

E_r -term completely incompressible from algebraic geom. pt of view.