

①

$$L_{k(z)} \text{ map} = \varprojlim_{a,b} \omega_2^{-1} \text{ map} / p^a, v_i, b$$

$$① \quad H(a, b) = (\mathbb{S}^0 \cup_{p^a} e^1) \cup_{v_i, b} C\Sigma^1 (\mathbb{S}^0 \cup_{p^a} e^1)$$

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Claim 1
For any X , $L_{k(z)} X = \varprojlim_{a,b} L_{k(z)} X \wedge H(a,b)$
true, because

smashing
w/ fin. spectra
commutes
with any
localization

$$L_{k(z)} X \rightarrow \varprojlim_{a,b} L_{k(z)} X \wedge H(a,b)$$

& both sides are $K(z)$ -local
need to show that the map is a $K(z)$ -equiv. F -type 2 finite complex.

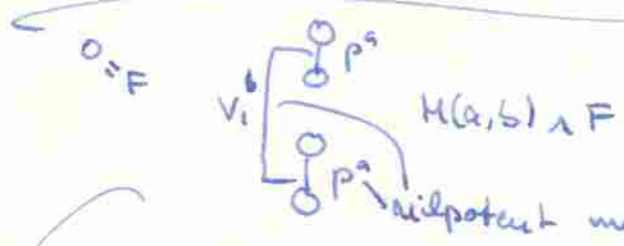
Künneth: suffices to show

$$X \wedge F \rightarrow \left(\varprojlim_{a,b} L_{k(z)} X \wedge H(a,b) \right) \wedge F$$

a $K(z)$ iso.

$$\varprojlim_{a,b} \left(L_{k(z)} X \wedge H(a,b) \wedge F \right)$$

smashing
w/ fin
comm
w/ \varprojlim



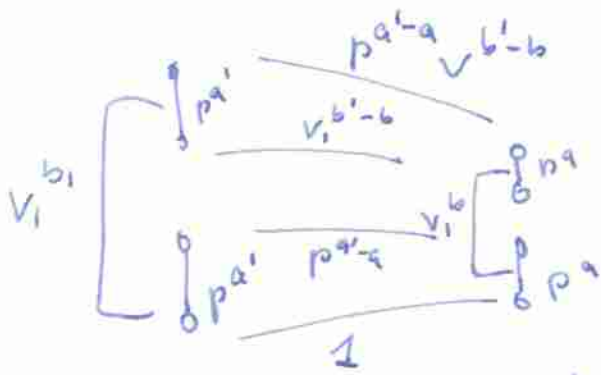
nilpotent maps in $F \Rightarrow \text{for } a, b \gg 0$

both maps are null and

$$\Rightarrow H(a,b) \wedge F = F \vee \Sigma F \vee \Sigma^{|v_i, b|+1} F \vee \Sigma^{|v_i|+2} F$$

Now for $a, b \gg 0$, what do the maps look like

②



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these will all become 0 for difference big enough

so let's replace our system with one where $d_{i,j}$ are zero (apart of course from 1)

$\leadsto \varinjlim =$ just the bottom copy

(i.e. colimit of ... \cong contractible).

proves $L_{K(2)} X = \varinjlim L_{K(2)} X \wedge M(a,b)$

$\Rightarrow = \varinjlim L_{K(2)} X \wedge v_2^{-1} M(a,b)$ \uparrow
 $v_2 = K(2)\text{-eq}$

Now suppose R is a BP-module, then

$R \wedge v_2^{-1} M(a,b)$ is $K(2)$ -local

reason

$\langle BP \wedge v_2^{-1} M(a,b) \rangle = \langle K(2) \rangle$

Module over ring spectrum is local over it b/c any acyclic map \rightarrow factors over a $R \wedge \dots$

Suppose $R \wedge A = a$ BP-module, A fin type 0

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$\Rightarrow R \wedge V_2^{-1} M(a, b)$ is $K(k)$ -local

$\{A \mid \text{with this property}\} = \text{thick}$

$R \wedge V_2^{-1} M(a, b) \wedge A$
is $(K(k))$ -local

$\Rightarrow \text{Lien } R = \varprojlim_{M(a,b)} V_2^{-1} R \wedge A$

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Now let's look at the case $\mathbb{F} = \text{truf}$
 $A = \mathbb{D}(A_1)$ then $R \wedge A = \text{BP}\langle 2 \rangle$

$$M_n \rightarrow L_n \rightarrow L_{n-1}$$

$(\mathbb{Z}/2$ versus $K(k))$

this can be understood
in terms of $K(k)$ -local theory

$$L_{K(k)} X = \varprojlim L_n X \wedge M(a_0 \dots a_{n-1})$$

$$M_n X = \varinjlim L_n X \wedge \mathbb{D} \rightarrow$$

Morava module

$A \quad \Gamma$

$$x' = \lambda^3 (x + \mathbb{F})$$

$$y' = \lambda^2 (y + \mathbb{S}x + t)$$

$\Gamma = A[\text{ris}_t] [\lambda^{\pm 1}]$

?

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λ should go in here
Let's take the following Hopf algebra

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$$\mathbb{Z} \rightrightarrows \mathbb{Z}[\lambda, \lambda^{-1}] \rightrightarrows \mathbb{Z}$$

G_m

claim: G_m -comodules \cong graded abelian groups

$$A \rightarrow A[\lambda, \lambda^{-1}]$$

$$A = \bigoplus_{n \in \mathbb{Z}} A_n$$

$$a_n \otimes \lambda^n \longleftrightarrow a_n \in A_n$$

conversely, (exercise) if A is a comodule,

$$A_n := \{a \mid \psi(a) = a \otimes \lambda^n\}$$

$$\Rightarrow A = \bigoplus A_n$$

Consequences: no higher cohomology

$$A_n = \text{Hom}_{\text{comod}}(\mathbb{Z}(n), A)$$

$$\mathbb{Z}(n) \longrightarrow \mathbb{Z}(n) \otimes \mathbb{Z}[\lambda, \lambda^{-1}]$$

$$1 \longmapsto 1 \otimes \lambda^n$$

\Rightarrow no higher Ext groups

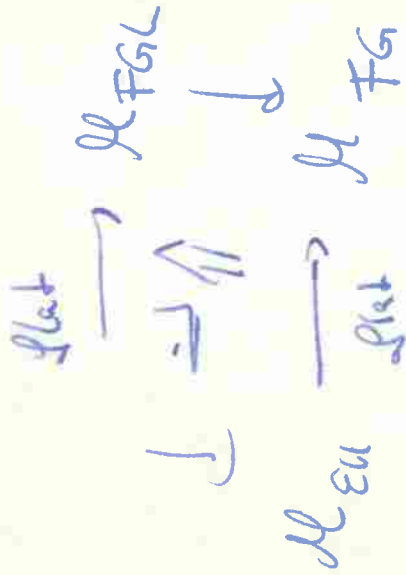
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Extension of Hopf algebras

$$A[t, s, t^{-1}] \rightarrow A[t, s, t^{-1}][\lambda^{\pm 1}] \rightarrow \mathbb{Z}[\lambda^{\pm 1}]$$

having λ in is equivalent to
not having it in & having
a grading.

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$$\text{turf} \rightarrow \text{turf} \wedge X(4) \xrightarrow{\text{MU-injection}} \text{turf} \wedge X(4) \wedge X(4) \dots$$

$$\text{turf} \wedge \text{MU} \xrightarrow{\text{is}} \text{turf} \wedge X(4) \wedge \text{MU}$$

$\text{turf} \wedge X(4) [x_5, x_6, \dots]$

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$$\text{turf} \wedge \text{MU} \rightarrow \text{turf} \wedge X(4) \wedge \text{MU}$$

exact: splits
 b/c

injective:

but $\text{turf} \wedge X(4)$
 is exact, \Rightarrow MU-
 algebras
 i.e. stacks of $\text{MU} \wedge \dots$

to show:
 this is MU-
 inj. resolution
 i.e. each inj .
 & exact after $\wedge \text{MU}$

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Suppose that J is a generalized elliptic curve

(1) over a ring $U = \text{Spec } R$.

Then J is classified by a map

$$(*) \quad U \rightarrow \mathcal{M}_{\text{ell}},$$

where \mathcal{M}_{ell} is the moduli stack of elliptic curves.

If the map $(*)$ is flat then the formal completion of J is a Landweber exact formal group over R , and so gives a cohomology theory, $E(J)$.

Theorem: There is a unique (up to weak equivalence) lift of E to a sheaf of E_∞ -ring spectra on the moduli stack of generalized elliptic curves in the "quasi-étale" topology. (without additive fibre!)

Let's call this sheaf \mathcal{E} . The spectrum turf is the (-1) connected cover of $E(\mathcal{M}_{\text{ell}})$:

$$\text{turf} = E(\mathcal{M}_{\text{ell}}^{\circ}) \langle 0 \dots \text{inf} \rangle.$$

This produces turf as an E_∞ ring spectrum.

The notes on $K(1)$ -local E_∞ -ring spectra give this at the primes 2 and 3. I haven't worked out the details at larger primes (where I think they are easier)

Generalized elliptic curve: genus one curve with a marked smooth point, with at worst nodes as singularities.

Mike's email quasi-étale topology: $\mathbb{R} \rightarrow S$ is quasi étale if
 (2) it is flat and if the relative cotangent complex is acyclic.

(2) Here are some ~~more~~ ^{other} things you might have heard: Let $X(4)$ denote the Thom spectrum of the map

$$\Omega SU(4) \rightarrow \Omega SU = BU.$$

Then $\text{turf} \wedge X(4)$

is complex orientable. In fact

$$\pi_* \text{turf} \wedge X(4) = \mathbb{Z}[a_1, a_2, a_3, a_4, a_6] = A,$$

and the formal group law is the one coming from the Weierstrass elliptic curve. Even more, the Hopf algebroid (A, Γ)

$$A = \pi_* (\text{turf} \wedge X(4))$$

$$\Gamma = \pi_* (\text{turf} \wedge X(4) \wedge X(4)) = A[r, s, t]$$

is just the Weierstrass Hopf algebroid, corresponding to the change of coordinates

$$x \mapsto x+r$$

$$y \mapsto y + sx + t$$

in the Weierstrass curve

$$y^2 + a_1 xy + a_3 y = x^3 + a_2 x^2 + a_4 x + a_6.$$

It follows that the $X(4)$ based Adams spectral sequence for turf (which can easily be shown to be the MU -based ASS) has E^2 -term the coho-

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mology of the Weierstrass Hopf-algebra.

These facts follow easily from the fact that one can identify

Weierstrass curve = an elliptic curve with a coordinate modulo degree 5.

There is a proof in that long unfinished paper "Elliptic Curves and Stable Homotopy".

(3) One can compute

$$H^*(turf; \mathbb{Z}/2) = A//A_2$$

where A_2 is the subalgebra of the mod 2 Steenrod algebra generated by Sq^1 , Sq^2 , and Sq^4 (you're supposed to be reminded of the computation

$$H^*(bo; \mathbb{Z}/2) = A//A_1.$$

It follows that, at the prime 2,

$$turf \wedge \mathbb{D} = BP\langle 2 \rangle,$$

where \mathbb{D} is a finite spectrum whose cohomology is "double of A_1 " = $A_2 // E[Q_0, Q_2]$.

This might characterize $turf$, but I'm not sure.

I hope these comments help.