Transitivity: $k \to A \to B$, M a B-module.

Then get long exact sequence

$$\cdots \; H^{s+1}(B/A, M) \leftarrow H^s(A/k, M) \leftarrow H^s(B/k, M) \leftarrow H^s(B/A, M) \leftarrow \cdots$$

16.10.03

Announcement: Γ-cohomology $\overset{\cong}{\longleftrightarrow} H^*_{\mathcal{E}(sMod_k)}(-)$

follows from work of Bastena–McCarthy, Mandell.

§1 Adams type spectra

E: homotopy commutative ring spectrum

(1) $E = \operatorname{hocolim}_\alpha E_\alpha$ with E_α finite CW (filtered)

(2) $E_* DE_\alpha$ is a projective E_*-module (D = Spanier-Whitehead dual)

(3) $[DE_\alpha, F] \longrightarrow \operatorname{Hom}_{E_*E}(E_* DE_\alpha, E_*F)$ is an iso

$\xrightarrow{\cong} \operatorname{Hom}_{E_*}(E_* DE_\alpha, F_*)$ for all E-module spectra F.

All Landweber exact spectra satisfy these.

Lemma: If $[\Sigma^n DE_\alpha, X] \xrightarrow{\cong} [\Sigma^n DE_\alpha, Y]$ for $f: X \to Y$ and all n, then $E_* f: E_* X \to E_* Y$ is also iso.

Pf: $E_n X = [S^n, E \wedge X] = \operatorname{colim}_\alpha [S^n, E_\alpha \wedge X]$

$= \operatorname{colim}_\alpha [\Sigma^n DE_\alpha, X]$

§2. <u>Stover resolutions</u> (Bousfield)

Let $\mathcal{P} = \{\Sigma^n DE_\alpha\}_{n,\alpha}$

- A map $f: X \to Y$ of spectra is <u>$\mathcal{P}$-epi</u> if

$$[P, X] \to [P, Y] \text{ for all } P \in \mathcal{P}.$$

- A spectrum Q is $\mathcal{P}$-projective if $[Q, X] \xrightarrow{f_*} [Q, Y]$ is epi for all $\mathcal{P}$-epis $f: X \to Y$

Note: $\underline{\mathcal{S}}$ has enough $\mathcal{P}$-projectives, namely

$$\left(\bigvee_{P \in \mathcal{P}} \bigvee_{f: P \to X} P \right) \longrightarrow X .$$

- $f: A \to B$ is a $\mathcal{P}$-projective cofibration if it has the LLP

$$\begin{tikzcd} A \arrow[r] \arrow[d, "f"'] & X \arrow[d] \\ B \arrow[r] \arrow[ur, dashed, "\exists"] & Y \end{tikzcd} \quad \text{fibration and } \mathcal{P}\text{-epi}$$

<u>Thm</u> (Dwyer-Kan-Stover, Goerss-Hopkins, Bousfield, Hirschhorn)

$s\underline{\mathcal{S}}$ is a simplicial model category where

(i) weak equivalences $= E_*$-equivalences $=$ $f: X_\bullet \to Y_\bullet$ such that $\pi_* E_* X \to \pi_* E_* Y$ is iso.

(ii) f is E_*-cofibration $\Leftrightarrow$

$$X_n \cup_{L_n X} L_n Y \longrightarrow Y_n \text{ is a } \mathcal{P}\text{proj. cofibration.}$$

$$L_n X = \text{"degeneracy"} = \text{latching object} = \operatorname*{colim}_{\substack{\varphi: [n] \twoheadrightarrow [k] \\ k < n}} X_k .$$

This is a localization of the "E_2-model structure" in which weak equivalences are those $f: X \to Y$ which induce isos on

$$\pi_* [\Sigma^n DE_\alpha, X_\bullet] \to \pi_* [\Sigma^n DE_\alpha, Y_\bullet] \text{ for all } \alpha, n$$