

# HOMEWORK 11: HOMOLOGY

PROBLEM 1: Let's practice quotients a bit. Let  $M^2\mathbb{R}$  be the space of  $2 \times 2$  matrices with entries in  $\mathbb{R}$ .

1. Observe that  $M^2\mathbb{R}$  is a vector space of dimension 4
2. Let  $I =$  subspace spanned by the matrix  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
3. Describe as best you can the quotient vector space  $M^2\mathbb{R}/I$

PROBLEM 2: Give examples of two spaces which have the same homology, yet are not homeomorphic.

EXTRA CREDIT: same homology, but not homotopic...

PROBLEM 3: Compute  $H_*(\mathbb{R}P^2; G)$  for  $G = \mathbb{Z}_2, \mathbb{R}, \mathbb{Z}$ .

PROBLEM 4: What is  $H_p$  for  $p: T^2 \rightarrow K^2$  the covering space (2-sheeted cover of torus-to-Klein)? Use  $\mathbb{Z}_2$  and  $\mathbb{Z}$  coefficients.

