HOMEWORK 9 π_1 & functoriality

PROBLEM 1: Use the Van Kampen theorem to compute $\pi_1(K^2)$ where K^2 is the Klein bottle.

PROBLEM 2: Give an example of a connected space X with $\pi_1 X \cong \mathbb{Z}$ but $X \not\simeq S^1$. (Hint: assume the as-yet-unproven result that Euler characteristic is a homotopy invariant.)

PROBLEM 3: If $i : A \hookrightarrow X$ is an injective (one-to-one) map, is $\pi_1 i : \pi_1 A \longrightarrow \pi_1 X$ an injective homomorphism?

PROBLEM 4: A <u>RETRACTION</u> of X to $A \subset X$ is a map $r : X \to A$ such that $r(a) = a \ \forall a \in A$. Use functoriality to show that $\pi_1 r : \pi_1 X \to \pi_1 A$ is surjective and $\pi_1 i : \pi_1 A \to \pi_1 X$ is injective.