R. GHRIST: STATEMENT ON RESEARCH

The theme of my research is **topological methods in applied mathematics**. This entails discovering new applications for topological tools, ideas, and methods in problems broadly construed as "applied." This latter category ranges from concrete physical problems (e.g., coverage in sensor networks or multiple robot coordination) to more abstract problems which are merely inspired by applications (e.g., hydrodynamics on Riemannian manifolds or bifurcation theory). Because I am most interested in a broad technology transfer from topology and geometry to engineering, I work across several disciplines within topology and its applications, collaborating frequently with the appropriate experts. The majority of my collaborative efforts are with researchers in Electrical/Systems Engineering, Robotics, and Computer Science.

I am evangelical in my belief that **topology** — the study of abstract spaces and their properties — is an ideal mathematical discipline for contemporary problems in science and engineering. Topologists have spent over one hundred years developing algebraic, differential, and geometric methods for characterizing spaces and continuous functions between them up to various coarse forms of equivalence. Topological results, being based not on precise distances or displacements but rather upon global features, tend to be very robust and insensitive to noise and various 'errors,' a useful feature for real-world problems. Unfortunately, all but the most trivial topological concepts and tools have remained largely ensconced within Mathematics departments, walled off from scientists and engineers by a formidable history of specialized terminology and subtle but non-intuitive algebraic constructs. My goal is to demonstrate forcefully the particular blend of efficacy and beauty that characterizes topological methods, and to communicate these methods to researchers in applied fields.

I have a dual goal in reforming Mathematics as a discipline to be more engaging with the scientific community at large and to work actively to take its rightful place as the Queen of Sciences.

RESEARCH RESULTS BY AREA

The following are short synopses.

1. Knots, Links, and Braids in Dynamical Systems:

1.A. My Ph.D. thesis (G, 1995) concerned knotted periodic orbits in 3-d vector fields. The principal result was the discovery of a structurally stable, real-analytic flow possessing closed orbits of **all possible knot and link types**, disproving a 1983 conjecture of Birman and Williams. The techniques involved developing a symbolic renormalization theory for **branched surfaces** with semiflows (G, 1997). See the 1997 monograph for a comprehensive treatment.

1.B. I demonstrated that flows with such knotting properties are not uncommon in ODE models arising in applications (G+Holmes, 1996), in bifurcations (G+Young, 1998), and in flows transverse to knot and link **fibrations** (G+Kin, 2004).

1.C. I analyzed and classified the knots which arise as singularities on a 3-sphere for gradient fields with a plane-field constraint (Etnyre+G, 1999).

1.D. I proved the existence of a smooth, steady, nonsingular **perfect fluid flow** on a Riemannian 3-sphere which exhibits closed particle paths of all knot and link types (Etnyre+G, 2000).

1.E. I created a **braid-theoretic Morse theory** for solutions to parabolic scalar lattice dynamics (G+van den Berg+Vandervorst, 2003). This is a relative Conley-type homotopy index that yields a braid-theoretic forcing theory for parabolic lattice dynamics. We used this **homotopy braid index** to prove very general forcing theorems for **second order Lagrangians**.

1.F. The above-mentioned homotopy index was extended to solutions of general **scalar parabolic PDEs** (G+Vandervorst, 2007). This theory is used to force the existence of infinitely many stationary or time-periodic solutions to parabolic PDEs given the braid class of a skeleton of stationary solutions.

2. Contact Topology and Fluid Dynamics:

2.A. In a series of papers (Etnyre+G, 1999,2000,2001,2002) a novel relationship between the flow of a perfect inviscid fluid flow and the **contact topology** of the orthogonal plane field in dimension three. This allows one to import the deep results of Hofer et al. on pseudoholomorphic curve techniques in contact dynamics (**Reeb fields**) to prove theorems about fluids. E.g., every sufficiently smooth steady inviscid flow on a Riemannian 3-sphere has a closed flowline which is furthermore unknotted.

2.B. A combination of ideas from contact topology and **spectral geometry** were used (G+Komendarczyk, 2006) to show that there are **L^2 (energy) minimizing** steady fluid flows on a 3-sphere whose transverse contact structures are **overtwisted**, contradicting an earlier conjecture.

2.C. As a capstone of this project, an application of the **contact homology** of Eliashberg, Givental, and Hofer was given to the perennial problem of **hydrodynamic instability** of 3-d inviscid fluid flows (Etnyre+G, 2006). We demonstrated that for a generic Riemannian metric on R^3 (with periodic boundary conditions), all curl-eigenfield solutions to the Euler equations are (linear, L2) hydrodynamically unstable.

3. Topology and Geometry in Robotics:

3.A. Motivated by the problem of robot motion planning on a graph (tracks, guidewires) I began classifying **configuration spaces of graphs** (G, 2001), eventually using topological results to detail motion-planning algorithms for use with **automated guided vehicles** (G+Koditschek, 2002).

3.B. The appropriate notion of a configuration space for systems which involve modular reconfiguration was developed, motivated by shape-changing **metamorphic robots** (Abrams + G, 2004). A classification of the geometry and topology of these **state complexes** was recently published (G+Peterson, 2007). This work shows that any reconfigurable system (independent of the engineering details) has a locally CAT(0) state complex. As such, path-optimization in these configuration spaces is especially parsimonious: no local minima of path-length.

3.C. An examination of the topology associated to **graph grammars** yielded results on **self-assembling** programmable devices, including a classification of communication requirements needed to generate guaranteed stable assemblies (Klavins+G+Lipsky, 2006). The results were proved using covering space theory.

3.D. I constructed a **coordination space** to solve certain problems in the collision-free coordination of multiple robots. By proving that such spaces are metric spaces of **nonpositive curvature**, a complete classification of (Pareto-) optimal coordinations was derived in terms of the topology of the coordination space (G+LaValle, 2006). This is useful in creating tables of optimal coordinations which are valid independent of the scalarization of the cost functional used. Ideas from **geometric group theory** give an implementable algorithm to compute these optimal 'normal forms' (G + O'Kane + LaValle, 2005).

3.E. Recent work demonstrates that techniques from the geometry of metric spaces with curvature bounded above by zero (**CAT(0)** spaces) give surprisingly general results in pursuit-evasion games of interest in robotics and defense applications (Alexander + Bishop + Ghrist, 2006, 2007 preprint).

4. Algebraic Topology and Sensor Networks:

4.A. A recent sequence of papers (de Silva + G, 2006, 2007) demonstrated that homology theory is extremely powerful in problems involving coverage and hole-detection in **sensor networks**. The core idea is to handle networks whose nodes are non-localized (no GPS, no distance measurements, no angular measurements, etc.) by completing the network communication graph to a simplicial complex and using the homology of this **Rips complex** to detect holes in coverage.

4.B Recent work (Chambers, de Silva, Erickson, + G, 2008 preprint) classifies Rips complexes of points in the plane in terms of the k-connectivity of the projection map to the plane. Results using persistence apply to systems with a quasi-unit-disc construction.

4.C. Simple topological ideas from degree theory were used (G+Lipsky+Poduri+Sukhatme, 2006; G, 2007) to build **surrounding cycles** given a node in a network communication graph. These techniques solve the problem of how to isolate an intruder who sets off an alarm without knowing where either the intruder or the sensors are.

4.D. In recent work, (Baryshnikov+G, 2007 preprint) we apply a topological integration theory arising in **constructible sheaves** to solve problems of data aggregation over a sensor network. This **integration with respect to Euler characteristic** has a number of surprising and useful properties for target counting without localization or target identification.

Future Research Goals

In the next four years, I intend to focus on the following projects, all of which fit under the aegis of *topological methods in applied mathematics*.

5. **Data Integration over Sensor Networks:** This project will apply integration ideas from sheaf theory (including the rich and rapidly-developing theory of motivic integration) to pass from local to global data in sensor networks and networked signal processing.

6. **Topological Statistics:** This project will adapt ideas from Lusternik-Schnirelmann category theory to a topological data analysis that focuses on general unimodal distributions and their topology, as opposed to rigid (Gaussian, beta, Bernoulli, etc.) distributions and their analysis.

7. **Programmable Matter:** This project will combine geometric group theory and distributed computational methods to perform high-level task planning in metamorphic and self-assembling systems.

8. **Floer Theory for Applied Differential Equations:** Much of the amazing progress in Floer theory and its offspring have found applications only in geometry and topology. This project will establish a toolbox of Floer-type results applicable to differential equations of interest in fluid dynamics, pattern formation, biology, and other applied fields.