Math 234

Section 1.1: This section covers the slope of a line. The material should treated as review.

$$slope\ of\ \ a\ line = m = \frac{y-y_0}{x-x_0} = \frac{\Delta y}{\Delta x} = \frac{rise}{run}$$

Two useful forms of the equation of a line:

1) Point-slope: $y - y_0 = m(x - x_0)$

2) Slope-Intercept: $y = mx + b$

where m is slope and (x_0, y_0) is a point on the line.

The greater the slope, the greater the line increases as x goes from left to right.

The more negative the slope, the greater the line decreases as x goes from left to right.

A slope of 0 is a horizontal line since there is no change in the height of the line y for a given change in x.

Sections 1.2 -1.3: Derivatives and Slope

Motivation: Two interesting questions.

1) If we know the position of an object at every instant, shouldn't we know its velocity at every instant?

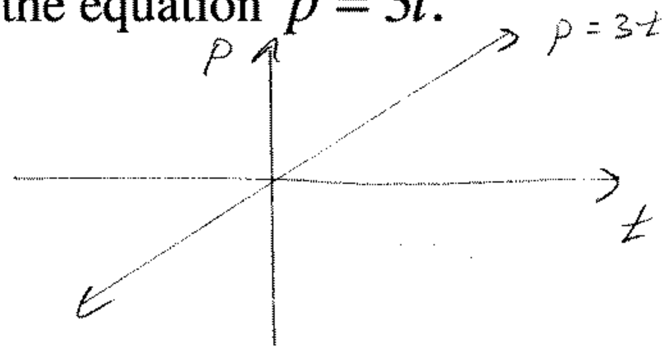
2) If we know the starting point of an object and its velocity at every instant, shouldn't we know its position at every instant?

These two questions led Isaac Newton in 1665 to develop calculus.

The first question leads to what is known as differential calculus, and is the motivating question for the first part of the course.

We know how to answer question 1 in the case that the speed is constant:

For example, suppose a car is traveling at 10 mph. Let t be time in hours and suppose the car starts traveling at time $t = 0$. Then its position at time t is given by the equation $p = 3t$.

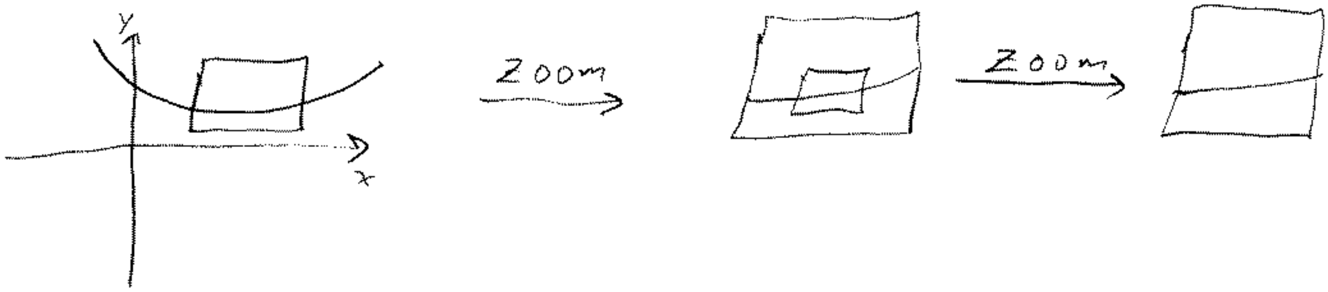


The velocity at a given time is just distance/time:

$$\frac{p_2 - p_1}{t_2 - t_1} = slope\ of\ the\ line = 3$$

So, the slope gives velocity when the position function is a line. But slope is defined only for lines. How can we extend the definition to more general curves?

Answer: Zoom in! Zoom in on the curve at a point until the curve looks like a line. Then the slope of that line will be the slope of our function at that point. If you have a scientific calculator, try this!



Note: We actually encounter this phenomenon everyday in three dimensions. The earth is round, but when we look around, it appears flat. That's because we've zoomed in – we see so little of the earth that we don't notice the curvature.

Definition: Given a function $f(x)$, the derivative of $f(x)$ at a point x is the formula for the *slope* of f at any point x.

Notation: The derivative of $f(x)$ is written as

$f'(x)$ or $\frac{d}{dx}f(x)$.

Some Basic Formulas for Derivatives:

1) $f(x)=b$ $\quad f'(x)=0.$

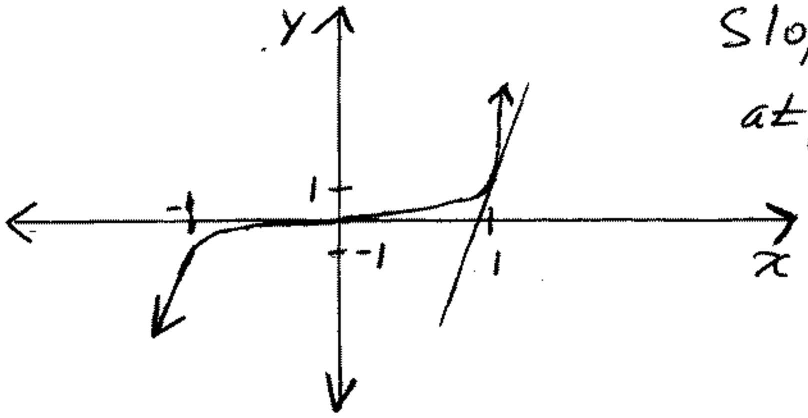
2) $f(x)=mx+b$ $\quad f'(x)=m.$

3) $f(x)=x^r$ $\quad f'(x)=rx^{r-1} \quad (r\neq 0)$

Example: (#4, page 91)

$$f(x) = x^{19} \qquad \text{so} \qquad f'(x) = 19x^{18}.$$

So, the slope of x^{19} at $x = 0$ is $f'(0) = 0$ and the slope of x^{19} at $x = 1$ is $f'(1) = 19$. Compare this to the graph of $f(x) = x^{19}$.



Slope of $f(x)=x^{19}$ at $x=1$ is $f'(1)=19$

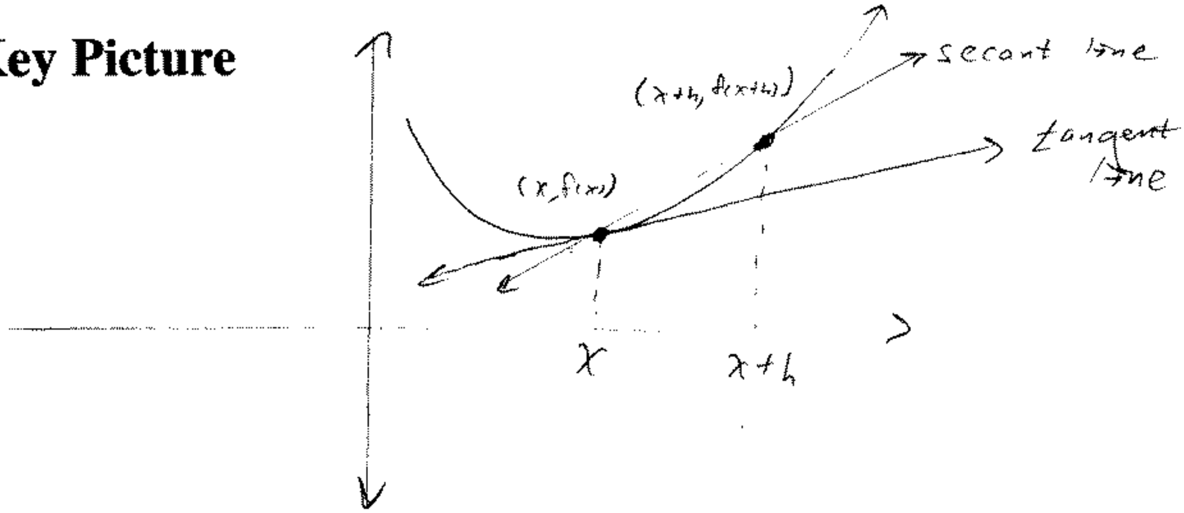
Example: (#6, page 91)

$$f(x) = \frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}} = x^{-1/2}.$$

Therefore $f'(x) = -\dfrac{1}{2}x^{-3/2}$.

That's great, but how did we get rule 3)? More generally, how do get formulas to compute slope in general? The answer comes from this

Key Picture



The tangent line just touching f at x is the line with slope $f'(x)$.

Observe that as $h \longrightarrow 0$, the secant lines approach the tangent line at x. Therefore, for small h,

$$f'(x) \approx \text{slope of secant line}$$

$$= \frac{f(x+h)-f(x)}{(x+h)-x}$$

$$= \frac{f(x+h)-f(x)}{h}.$$

Example: (#52, page 93).

As $h \to 0$, what is the approximate value of $\frac{\sqrt{4+h}-\sqrt{4}}{h}$?

This is the above formula with $f(x)=\sqrt{x}$ at $x=4$.

$f(x)=\sqrt{x}=x^{1/2}$.

Therefore $f'(x)=\frac{1}{2}x^{-1/2.}$

Hence $\frac{\sqrt{4+h}-\sqrt{4}}{h} \approx f'(4)=\frac{1}{2}\frac{1}{\sqrt{4}}=\frac{1}{4}$.

Let's take this one step further and calculate the tangent line to $f(x)=\sqrt{x}$ at $x=4$.

Use point-slope form of the equation of a line:

$y-y_0=m(x-x_0)$ where

$(x_0,y_0)=(4,\sqrt{4})=(4,2)$ and $m=f'(4)=\dfrac{1}{4}$

Hence the tangent line to $f(x)=\sqrt{x}$ at $x=4$ is:

$$y-2=\frac{1}{4}(x-4).$$