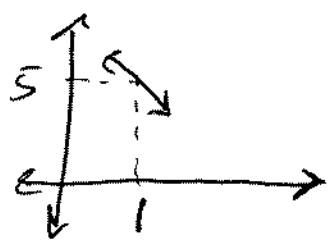
Sections 2.3 – 2.4: Curve Sketching

Goal: To use information from derivatives to understand the shape of a curve.

Example: (#14, page 206) Suppose $g(1)=5, \quad g'(1)=-1$. Draw some conclusion about $g(x)$ near $x=1$ and sketch it.

$g'(1)=-1<0$ means $g(x)$ is decreasing (with slope -1) at $x=1$.



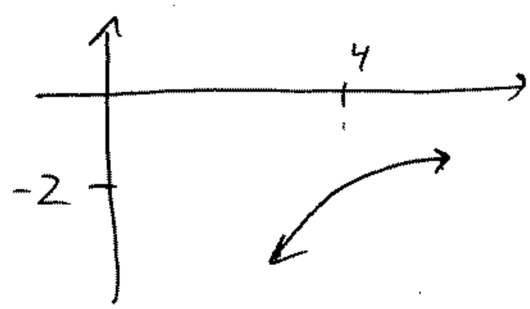
$g(1)=5$

$g'(1)=-1$

Example: (#18, page 206) Suppose $f(4) = -2$, $f'(4) > 0$, $f''(4) = -1$. Draw some conclusions about $f(x)$ near $x = 4$ and sketch it.

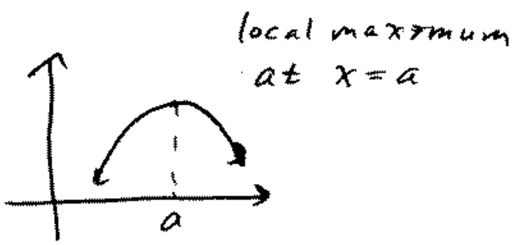
$f'(4) > 0$ means that $f(x)$ is increasing at $x = 4$.

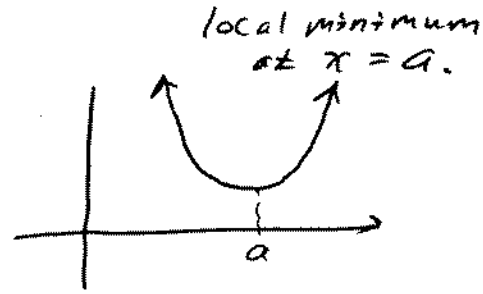
$f''(4) = -1$ means that $f(x)$ is concave down at $x = 4$.



$f(4) = -2$
$f'(4) > 0.$
$f''(4) = -1$

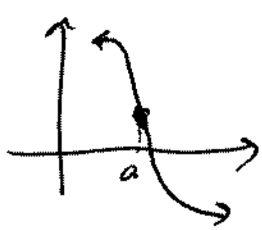
Observe from the pictures below that if $f(x)$ has a *local extreme point* (a *local minimum* or a *local maximum*) at $x = a$, then the tangent line at $x = a$ has slope zero. In other words, $f'(a) = 0$.

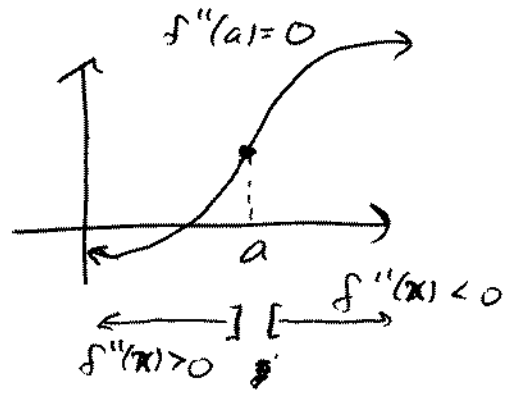




So:
To locate possible local extreme points for a function $f(x)$, check the points where $f'(x) = 0$.

At an inflection point, $f(x)$ changes from concave up $(f''(x) > 0)$ to concave down $(f''(x) < 0)$ or from concave down $(f''(x) < 0)$ to concave up $(f''(x) > 0)$. Hence an inflection point can occur only where $f''(x) = 0$.





So:
To locate possible inflection points for a function $f(x)$, find the points where $f''(x) = 0$.

Example: (#2, page 164) The function $f(x) = 3x^2 + 6x - 5$ has one relative extreme point. Find it, plot it, and check the concavity at that point. Use this information to sketch the graph of $f(x)$ near the extreme point.

Extreme Point: Find where $f'(x) = 0$.

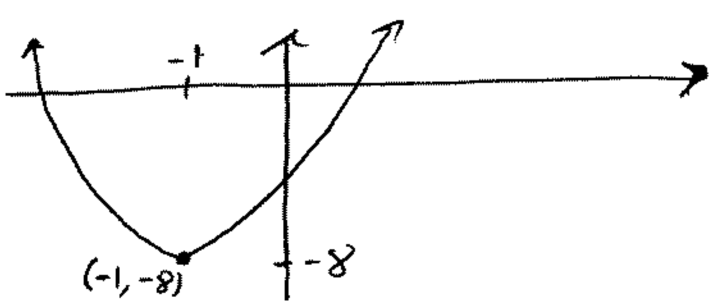
$f'(x) = 6x + 6$

$6x + 6 = 0 \quad 6x = -6 \quad x = -1$

Concavity: $f''(x) = 6$

Hence $f''(-1) > 0$, so that $f(x)$ is concave up at $x = -1$.

We conclude that near $x = -1$ $f(x)$ looks like:



We observe that the point $(-1, f(-1)) = (-1,-8)$ is a local minimum for $f(x)$.

Example: Find all inflection points for

$$f(x) = x^3 - 6x^2 + 19.$$

Solution: Find all *possible* inflection points by finding where $f''(x) = 0$.

$$f'(x) = 3x^2 - 12x$$

$$f''(x) = 6x - 12$$

So $f''(x) = 0$ if and only if $6x - 12 = 0$

if and only if $x = 2$.

Hence, the point $(2, f(2)) = (2,3)$ is the only *possible* inflection point for $f(x)$. It *is* an inflection point only if $f(x)$ changes concavity at $x = 2$.
Observe that

$$f''(x) = 6x - 12 < 0 \text{ if } x < 2$$

and

$$f''(x) = 6x - 12 > 0 \text{ if } x > 2.$$

Hence $f(x)$ changes from concave down to concave up at $x = 2$. Therefore, the point (2,3) is the only inflection point for $f(x)$.