

Review Questions for Exam 3

Things you should know or be able to do:

- 1) Evaluate indefinite integrals (basic rules, techniques of integration by parts and u-substitution).
- 2) Fundamental Theorem of Calculus.
- 3) Use of indefinite integrals to find definite integrals.
- 4) Applications of definite integrals
 - a) Area under a curve
 - b) Area between curves
 - c) Average Value of a function over an interval
 - d) Consumers' and Producers' Surplus
 - e) Volumes of Surfaces of Revolution
- 5) Improper Integrals. Determine if they are convergent or divergent, and evaluate them if they are convergent.
- 6) Know the definition of the probability density function and how to use it to compute probabilities.
- 7) Know how to compute first and second order partial derivatives.

Remember: You can check your integration work yourself by differentiating!

1) Evaluate the following indefinite integrals. Don't forget to write the "+C".

a) $\int (x^3 + x - 7) dx$

b) $\int \left(\sqrt{t} + \frac{1}{t} \right) dt$

$$\text{c) } \int (5-2t)^4 + e^{3t} dt$$

$$\text{d) } \int \frac{3x^2}{\sqrt{x^3+7}} dx$$

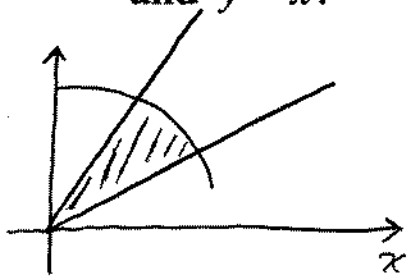
2) Compute the following:

a) $\int_0^1 xe^x dx$

b) $\int_1^3 x^2 e^{x^3} dx$

c) The area under the curve $y = \frac{\ln(x)}{x}$ from $x = 1$ to $x = e$.

3) Find the area shown bounded by $y = 12 - x^2$, $y = 4x$,
and $y = x$.



4) a) Let X be a continuous random variable with values between $A = 1$ and $B = \infty$, and with the density function $f(x) = 4x^{-5}$. Is $f(x)$ a probability density function for $x \geq 1$? Explain your work.

b) Compute $\Pr(4 < X < 6)$ for the continuous random variable X with probability density function $f(x) = 0.2e^{-0.2x}$.

5) a) Suppose the demand curve for some commodity is given by $p = x^2 - 14x + 49$. Find the consumers' surplus for $x = 10$.

b) Find the volume of the solid of revolution $y = \frac{k}{x^2}$ ^{generated by}
from $x = 1$ to $x = \infty$.

c) What values of k in part b) will make the area of the solid of revolution equal to 1?

5) Suppose that a colony of honey bees is growing exponentially with growth constant k . If there are currently 30,000 honey bees present, what will the *average* population of the colony be over the next six months? (Measure time in years.)

6) Compute the following:

a) *All* first order partial derivatives of

$$f(x, y) = \ln(xy + y^2) + ye^y$$

b) $\frac{\partial^2 f}{\partial y \partial x}$ for the Cobb-Douglas productivity function

$$f(x, y) = 12x^{1/3}y^{2/3} .$$

7) Determine whether the area under the curve

$y = x^2 e^{-x^3}$ over the interval $[1, \infty)$ is finite or infinite.

Compute the area if it is finite. Explain your work.

8) Find the average value of the function $f(x) = 8x(x+4)^{-3}$ on the interval $[0,4]$.