

You must show all your work to receive credit.

1a.] (3 pts) Find all the "critical numbers" of $f(x) = x^3 + \frac{3}{2}x^2 - 36x + 10$.

$$\begin{aligned} f'(x) &= 3x^2 + \frac{3}{2}(2)x - 36 \\ &= 3x^2 + 3x - 36. \end{aligned}$$

f' is never undefined.

f' is zero when $3x^2 + 3x - 36 = 0$

$$3(x^2 + x - 12) = 0$$

$$3(x+4)(x-3) = 0$$

$$3=0 \text{ or } x+4=0 \text{ or } x-3=0$$

$$\begin{array}{ccc} \times & x = -4 & x = 3 \end{array}$$

1b.] (2 pts) Find all the "possible inflection points" of $f(x)$.

$$f''(x) = 6x + 3.$$

f'' is never undefined.

f'' is zero when $6x + 3 = 0$

$$3(2x+1) = 0$$

$$3=0 \text{ or } 2x+1=0$$

$$\begin{array}{ccc} \times & 2x = -1 & \\ & x = -\frac{1}{2} & \end{array}$$

2 1c.] (3pts) Determine all intervals on which $f(x)$ is concave up and concave down.

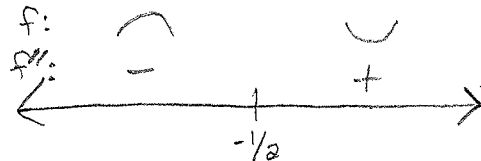


$$f''(-1) = 6(-1) + 3 = -6 + 3 = -3 < 0$$

So $f(x)$ is concave down on the interval $(-\infty, -\frac{1}{2})$.

$$f''(0) = 6(0) + 3 = 0 + 3 = 3 > 0$$

So $f(x)$ is concave up on the interval $(-\frac{1}{2}, \infty)$.



1d.] (2pts) Classify each critical number found in part 1a as a relative maximum, relative minimum, or neither.

Using the second derivative test:

$$f''(-4) = 6(-4) + 3 = -24 + 3 = -21 < 0$$



So $f(x)$ has a relative maximum at $x = -4$.

$$f''(3) = 6(3) + 3 = 18 + 3 = 21 > 0$$



So $f(x)$ has a relative minimum at $x = 3$.