Math 234 BD4 Quiz 7 (3.3, 3.4, 3.5) **Name (Last, First):** Solutions

You must show all your work to receive credit.

1.] (2 pts) At the price $p = 5$, determine if $D(p)$ (specified below) is elastic, inelastic, or is of unit elasticity. You may use the formula for $E(p)$ given below:

$$E(p) = \frac{p}{q} \cdot \frac{dq}{dp} \qquad D(p) = 100 - p^3$$

$D'(p) = -3p^2$

$D'(5) = -3(5)^2 = -75$

$q = D(p)$

$\frac{dq}{dp} = D'(p)$

$D(5) = 100 - 5^3 = 100 - 125 = -25.$

$E(p) = \frac{5}{D(5)} \cdot D'(5)$

$= \frac{5}{-25}(-75)$

$= 5 \cdot 3 = 15.$

$|E(p)| = |15| = 15 > 1$, So $D(p)$ is elastic at $p = 5$.

2a.] (1 pt) Find the domain of

$$f(x) = \frac{x^2 - 9}{x^2 + 3}$$

Since $x^2 + 3 = 0$ ($x^2 = -3$) has no solution,

The domain is all real numbers

2b.] (2 pts) For the same $f(x)$ as in part 2a, find the y-intercept and x-intercepts (if they exist).

x-int: $0 = \frac{x^2-9}{x^2+3}$

$0 = x^2 - 9$

$0 = (x+3)(x-3)$

$0 = x+3$ or $0 = x-3$

$-3 = x$ or $3 = x$

x-intercepts are

$(-3, 0)$ and $(3, 0)$

y-int: $f(0) = \frac{0-9}{0+3} = \frac{-9}{3} = -3.$

the y-intercept is

$(0, -3).$

2c.] (2 pts) Find the critical points of $f(x)$, and determine the intervals on which $f(x)$ is increasing or decreasing.

$$f'(x) = \left(\frac{x^2-9}{x^2+3}\right)' = \frac{2x(x^2+3) - (x^2-9)2x}{(x^2+3)^2} = \frac{2x[x^2+3-x^2+9]}{(x^2+3)^2}$$

$$= \frac{24x}{(x^2+3)^2}.$$

$f'(x)$ is never undefined because $x^2+3=0$ has no solution.

$f'(x)$ is zero when $-\frac{24x}{(x^2+3)^2} = 0$

$-24x = 0$

$x = 0.$

So the only critical point is $(0, f(0)) = (0,-3).$

2d.] (2 pts) Find all of the vertical and horizontal asymptotes of $f(x)$ (if they exist).

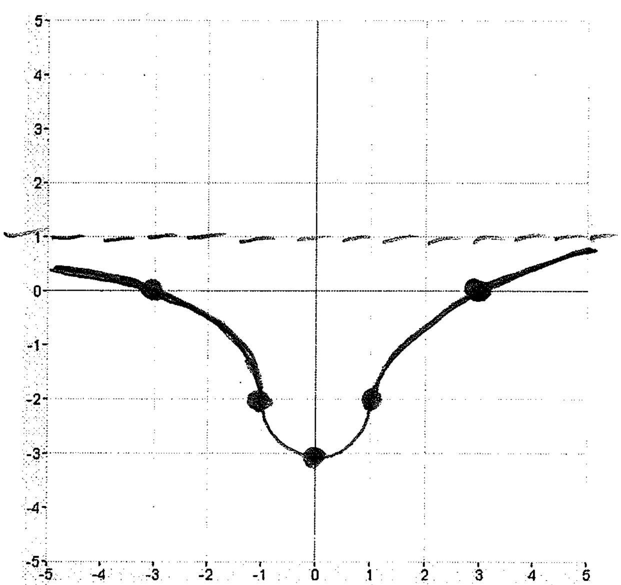
vertical asymptotes occur when $f'(x)$ is undefined (f has vertical slope) but the denominator of f' is never zero, as shown before.

So there are no vert. asyms,

horiz asymps? $\lim_{x\to\infty} \frac{x^2-9}{x^2+3} = \lim_{x\to\infty} \frac{1-9/x^2}{1+3/x^2} = \frac{1-0}{1+0} = 1.$

$\lim_{x\to\infty} \frac{x^2-9}{x^2+3} = \lim_{x\to-\infty} \frac{1-9/x^2}{1+3/x^2} = \frac{1-0}{1+0} = 1.$ So $y=1$ is a horiz asymp at both $x\to+\infty$ and $x\to-\infty$

2e.] (1 pts) Given: $f(x)$ is concave down on the intervals $(-\infty, -1)$ and $(1, \infty)$; $f(x)$ is concave up on the interval $(-1, 1)$. Use this information and your answers for parts a,b,c to (neatly) sketch the graph of $f(x)$. Include critical points, inflection points, intercepts and asymptotes (if any exist).



We are told that $x=1$ and $x=-1$ give inflection points: $(1, f(1)) = \left(1, \frac{1-9}{1+3}\right) = (1,-2).$

and $(-1, f(-1)) = \left(-1, \frac{1-9}{1+3}\right) = (-1,-2).$