Math 234 B Discussion BD6 Quiz 3 **Name (Last,First):** Solutions

Use the *definition* of the deriviative. You may *not* use any advanced differentiation techniques. You must show all your work to receive credit.

1.] (a.)(2pts) Compute $f'(x)$ (the derivative of $f(x)$), if $f(x) = \frac{4}{x}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{4}{x+h} - \frac{4}{x}}{h} = \lim_{h\to 0} \frac{\frac{4(x)(x+h)}{x+h} - \frac{4(x)(x+h)}{x}}{h}$$

$$= \lim_{h\to 0} \frac{4x-4(x+h)}{h(x+h)x} = \lim_{h\to 0} \frac{-4h}{xh(x+h)} = \lim_{h\to 0} \frac{-4}{x(x+h)} = \frac{-4}{x(x+0)} = \frac{-4}{x^2}.$$

(b.)(1pts) Find the slope of the line that is tangent to the graph of $f(x)$ at $x = -3$.

$$f'(-3) = \frac{-4}{(-3)^2} = -\frac{4}{9}.$$

2.] (a.)(2pts) Compute the derivative of the function $f(x) = 3 - 5x$.

$$f'(x) = \lim_{h\to 0} \frac{3-5(x+h) - \boxed{3-5x}}{h} = \lim_{h\to 0} \frac{3-5x-5h-3+5x}{h}$$

$$= \lim_{h\to 0} \frac{-5h}{h} = \lim_{h\to 0} -5 = -5.$$

(b.)(2pts) Find the *equation* of the line that is tangent to the graph of $f(x)$ at the point $\left(\frac{1}{5}, f\left(\frac{1}{5}\right)\right)$. Does your answer confuse you?

$m_{tan} = f'(\frac{1}{5}) = -5.$

$f(\frac{1}{5}) = 3 - 5(\frac{1}{5}) = 2.$

$y = mx+b$

$2 = -5(\frac{1}{5}) + b$

$3 = b.$

So $y = -5x+3.$

This makes sense because the tangent line of a line is that same line.

3.] (3pts) Find the rate of change $\frac{dy}{dx}$ at $x = 6$ for $y = 5x^2 - 20x + 1$.

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{5(x+h)^2 - 20(x+h) + 1 - [5x^2 - 20x + 1]}{h}$$

$$= \lim_{h\to 0} \frac{5x^2 + 10xh + 5h^2 - 20x - 20h + 1 - 5x^2 + 20x - 1}{h}$$

$$= \lim_{h\to 0} \frac{10xh + 5h^2 - 20h}{h} = \lim_{h\to 0} 10x + 5h - 20 = 10x - 20.$$

Then $\left.\frac{dy}{dx}\right|_{x=6} = 10(6) - 20 = \boxed{40}.$