

You must show all your work to receive credit.

1.] (2 pts) Differentiate the function; simplify your answer.

$$h(x) = (4x - 3)^{-1/2}$$

$$h'(x) = -\frac{1}{2}(4x-3)^{-\frac{1}{2}-1} (4x-3)' = -\frac{1}{2}(4x-3)^{-3/2} \cdot (4)$$

$$= -2(4x-3)^{-3/2} \quad \text{or} \quad \frac{-2}{\sqrt{(4x-3)^3}}$$

2.] (3 pts) Differentiate the function; fully simplify your answer (so that the numerator and denominator have no common factors).

$$f(x) = \frac{2x+5}{(1-2x)^3}$$

$$f'(x) = \frac{(2x+5)'(1-2x)^3 - (2x+5)[(1-2x)^3]'}{[(1-2x)^3]^2}$$

$$= \frac{2(1-2x)^3 - (2x+5)[3(1-2x)^2(1-2x)']}{(1-2x)^6} = \frac{2(1-2x)^3 - (2x+5)(3)(1-2x)^2(-2)}{(1-2x)^6}$$

$$= \frac{2(1-2x) - (2x+5)(-6)}{(1-2x)^4} = \frac{8x+32}{(1-2x)^4}$$

3.] (2pts) Does the tangent line to the graph of $f(x)$ ever become horizontal? Justify your answer. If 'yes', give some x value which makes the tangent line horizontal.

$$f(x) = (1 + \sqrt{x^3})^4 \quad f'(x) = 4(1 + x^{3/2})^3 (1 + x^{3/2})' = 4(1 + x^{3/2})^3 \left(\frac{3}{2}x^{1/2}\right)$$

$$= 6x^{1/2}(1 + x^{3/2})^3$$

Yes, $x=0$ makes $f'(0) = 6(0)(1+0)^3 = 0$.

4.] (3pts) The equation below describes a curve; use implicit differentiation to find the slope of the tangent line at the point $(1, \frac{1}{2})$. (Remember that y is a function of x)

$$xy + \frac{1}{y} = x$$

We want y' evaluated at $(1, \frac{1}{2})$: $x=1, y=\frac{1}{2}$. First, find y'

$$(xy + \frac{1}{y})' = x'$$

$$(xy)' + (\frac{1}{y})' = 1$$

$$(xy' + x'y) + (-y^{-2}y') = 1$$

$$xy' + 1y - y^{-2}y' = 1$$

$$xy' - y^{-2}y' = 1 - y$$

$$(x - \frac{1}{y^2})y' = 1 - y$$

$$y' = \frac{1-y}{x - \frac{1}{y^2}}$$

$$y' \Big|_{(1, \frac{1}{2})} = \frac{1 - (\frac{1}{2})}{(1) - \frac{1}{(\frac{1}{2})^2}}$$

$$= \frac{\frac{1}{2}}{1-4} = \frac{1}{2 \cdot (-3)}$$

$$= \left[-\frac{1}{6}\right] = m_{\tan}$$