



Math 220-X8

Test 1

Name _____

True False False [2 pts each]. *If False, you must explain why.*

- 1) For a linear function $f[x] = a x + b$, the growth rate of $f[x]$ is constant.
- 2) For a power function $f[x] = a x^k$, the growth rate of $f[x]$ is constant.
- 3) For an exponential function $f[x] = a e^{r x}$, the growth rate of $f[x]$ is constant.
- 4) On a global scale, exponential growth always dominates power growth because the percentage growth rate of a power function dies off to zero, but the percentage growth rate of an exponential function remains constant.
- 5) A good global scale plot of $f[x] = x^8 + 5 e^{-x}$ will look like the plot of $5 e^{-x}$.
- 6) The function $f[x] = \frac{4x+1}{x^4-17}$ will decay exponentially as x increases toward infinity.
- 7) $\text{Log}[e^{-2}] = -2$, but $e^{\text{Log}[-2]}$ does not equal -2 .
- 8) If we want to find the high points (crests, hills) and low points (dips, valleys) of a function $f[x]$, then we should start by solving the equation $f'[x] = 0$.
- 9) The function e^{-x} is always positive (i.e. the graph is always above the x -axis).
- 10) For all values of x , $e^x > e^{-x}$.
- 11) Underwater, the intensity of a light beam will decay at a constant rate, along the direction of the beam.
- 12) If $h = 0.1$, then $\frac{f[x+h] - f[x]}{h}$ measures the instantaneous growth rate of $f[x]$.
- 13) The function $f'[x]$ measures the instantaneous growth rate of $f[x]$.
- 14) If $f'[a] < 0$, then it is not possible that $f[a] \geq f[x]$ for all other x 's.

□ **Multiple Choice [2 pts each]. Choose the letter of the best answer.**

- 14) As h approaches 0, the graph of $\frac{\sin[x+h] - \sin[x]}{h}$ will look more and more similar to the graph of _____ .
 A) $\sin[x]$ B) $\cos[x]$ C) $\frac{\cos[x]}{h}$ D) $\sin[h]$
- 15) As h approaches zero, the plot of $\frac{e^{x+h} - e^x}{h}$ will look more and more similar to the plot of _____ .
 A) $\sin[x]$ B) $\frac{1}{h}$ C) e^x D) $\log[x]$
- 16) If $f'[-3] = 15$, then for values of x which are slightly to the right of -3 , we know that _____ .
 A) $f[-3] < f[x]$ B) $f[-3] > f[x]$ C) $f[3]$ is a maximum D) $f'[3] = f'[x]$
- 17) If $f'[-3] = 15$, then for values of x which are slightly to the left of -3 , we know that _____ .
 A) $f[-3] < f[x]$ B) $f[-3] > f[x]$ C) $f[3]$ is a minimum D) $f'[3] = f'[x]$
- 18) If $f[x] = 3e^{\log[1.4]x}$, then as x increases by 1 unit, $f[x]$ will increase by _____ .
 A) 3 % B) 30 % C) 40 % D) 140 %
- 19) _____ is an exponential function going through $(0, -2)$ and increasing by 80 % each time x increases 1 unit.
 A) $\log[80] e^{-2x}$ B) $-2e^{80x}$ C) $2e^{\log[1.8]x}$ D) $-2e^{\log[1.8]x}$

□ **Short Answer.**

20) [4 pts] Rank these in order of dominance (largest first and smallest last), as x approaches infinity :

$$e^{0.2x} \quad x^{0.2} \quad \sqrt{x} \quad e^{-0.2x} \quad 0.2e^x \quad e^{0.2} \quad x^2 \quad 0.2x$$

_____ .

Bonus) [2 pts] Rank these in order of dominance as x approaches infinity (largest to smallest), and explain how you ranked x^x : e^e x^x x^e e^x

21) [2 pts] Give a real-world situation that is modeled by exponential decay:

□ Limits [2 pts each]. What is the limiting value?

Show some steps or explain your answer.

$$22) \lim_{x \rightarrow \infty} \frac{3x^3 - 4x^2 + 2}{6x^3 + 5x}$$

$$23) \lim_{x \rightarrow \infty} \frac{15x^{12} + 2e^{3x}}{e^{3x} - x^2}$$

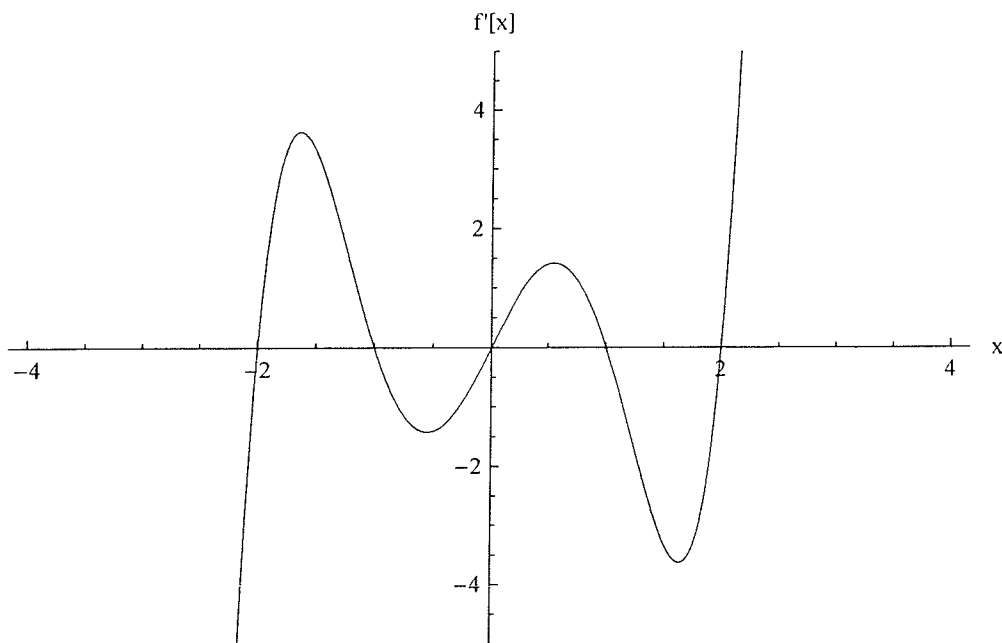
$$24) \lim_{x \rightarrow \infty} \frac{4x^9}{2 + e^{0.02x}}$$

$$25) \lim_{x \rightarrow \infty} 5e^{-x}$$

Bonus)
$$\lim_{x \rightarrow \infty} \frac{e^{-x} - e^{-4x}}{4e^{-4x} + 7e^{-x}}$$

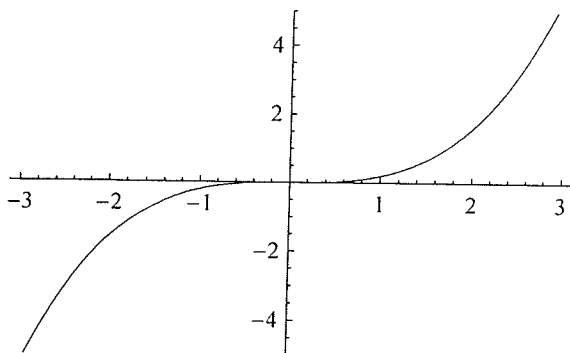
□ Graphs.

26) [4 pts] For some function $f[x]$, the graph of $f'[x]$ is shown below. List four values of x at which $f[x]$ might have a maximum (crest) or minimum (dip): _____

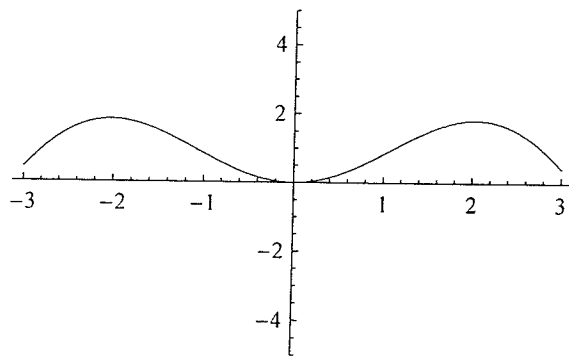


(27-29) Matching [2 pts each]. You are given the graph of $f[x]$. Write the letter for $f'[x]$ in the blank.

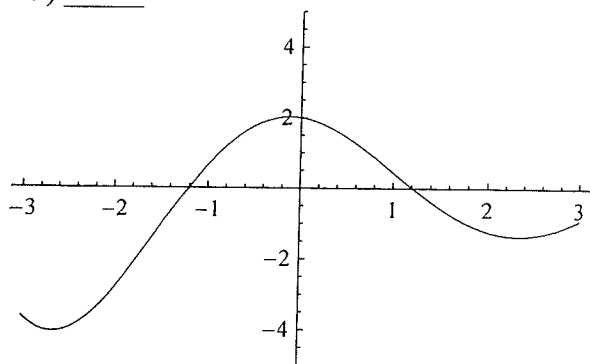
27) _____



28) _____

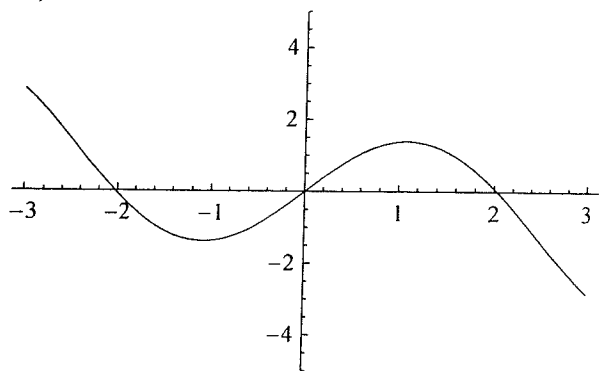


29) _____

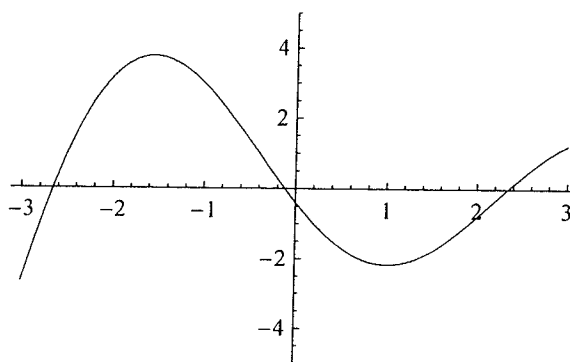


Choose $f'[x]$ from these :

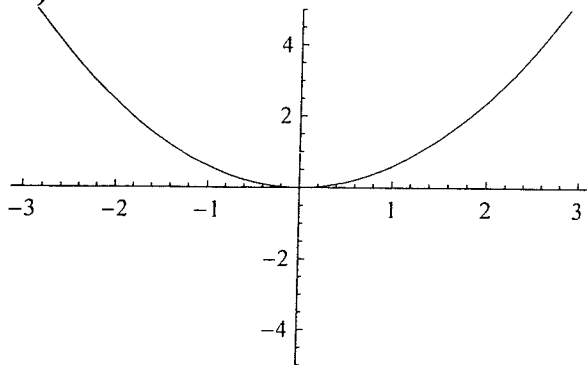
A)



B)



C)



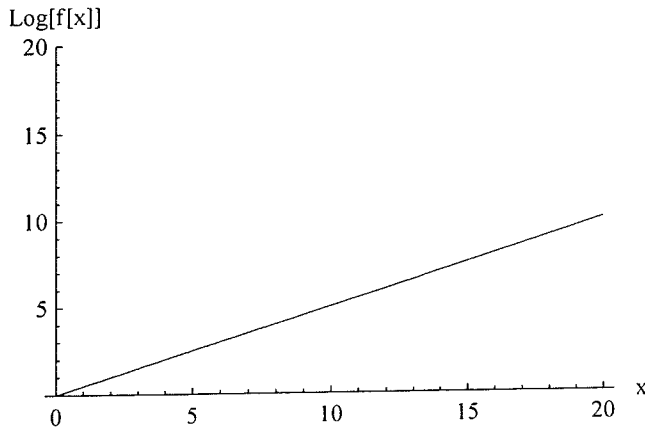
(30-34) [2 pts each].

30) Explain your answer to #27 :

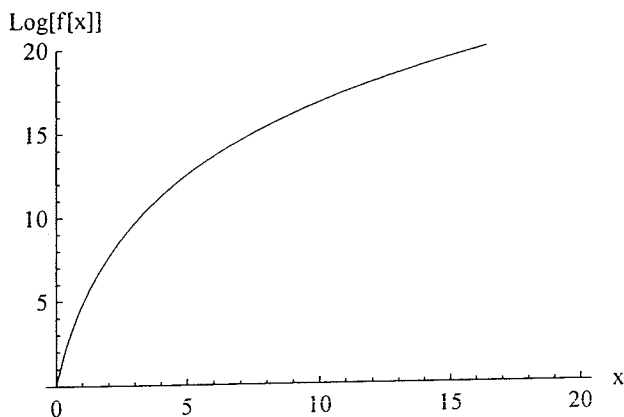
31) Explain your answer to #28 :

32) Explain your answer to #29 :

33) For some function $f[x]$, the graph of $\text{Log}[f[x]]$ is shown below. Is $f[x]$ growing exponentially? Why or why not?



34) For some function $f[x]$, the graph of $\text{Log}[f[x]]$ is shown below. Is $f[x]$ growing exponentially? Why or why not?



Bonus) Choose one of the two graphs above (either 33 or 34) and determine an explicit formula for $f[x]$, given that $f[0] = 2$.