

99/92

Math 220-X8

Test 2

Name Solutions

6 □ Calculate the Derivative [1 pt each].

$$1) f[x] = 3x^3$$

$$f'(x) = 9x^2.$$

$$2) f[t] = \frac{2}{t^2} = 2t^{-2}$$

$$f'(t) = -4t^{-3}.$$

$$3) f[x] = 5 \sin[x] - 7x$$

$$f'(x) = 5 \cos(x) - 7.$$

$$4) f[x] = -\cos[x] + e^x$$

$$f'(x) = \sin(x) + e^x.$$

$$5) f[t] = \sqrt{t} = t^{1/2}$$

$$f'(t) = \frac{1}{2} t^{-1/2}.$$

$$6) f[x] = 5e^{x/5}$$

$$f'(x) = 5 \cdot \frac{1}{5} \cdot e^{x/5} = e^{x/5}.$$

12 □ Calculate the Derivative [2 pts each]. Show some steps.

$$7) f[x] = -\sin[3e^{5x^2}]$$

$$f'(x) = -\cos[3e^{5x^2}] (3e^{5x^2})'$$

$$= -\cos[3e^{5x^2}] (3)(e^{5x^2})(10x)$$

$$= -30x e^{5x^2} \cos(3e^{5x^2}).$$

$$8) f[y] = \frac{e^y}{y^8} = e^y y^{-8}$$

$$f'(y) = e^y y^{-8} + e^y (-8y^{-9}).$$

$$9) f[t] = e^{-t} \sin[4t]$$

$$f'(t) = -e^{-t} \sin[4t] + e^{-t} \cos[4t] \cdot 4.$$

$$10) f[x] = \log[9x^2 - \log[x]]$$

$$f'(x) = \frac{(9x^2 - \log[x])'}{9x^2 - \log x} = \frac{18x - \frac{1}{x}}{9x^2 - \log x}$$

$$11) f[x] = \frac{(1-8x-x^2)^3}{1+x}$$

$$f'(x) = 3(1-8x-x^2)^2 (-8-2x)(1+x)^{-1}$$

$$+ (-1)(1+x)^{-2} (1-8x-x^2)^3.$$

$$12) f[x] = \frac{e^{ax}}{1-e^{ax}}$$

$$f'(x) = ae^{ax} (1-e^{ax})^{-1}$$

$$+ e^{ax} (-1)(1-e^{ax})^{-2} \cdot (-e^{ax})$$

$$= \frac{ae^{ax}}{1-e^{ax}} + \frac{ae^{2ax}}{(1-e^{ax})^2}.$$

□ True False False [2 pts each]. If False, you must explain why.

$$13) (5x)' = 0 \text{ because } 5'x' = 0 \cdot 1 = 0. \text{ False.}$$

$$(5x)' = 5(x)' = 5 \cdot 1 = 5.$$

$$14) \text{ Polynomial curves have a distinctive "s" shape. False.}$$

Logistic curves have the distinctive "S" shape.

15) If  $f'(a) = 0$ , then  $f(x)$  has a max or a min at the point  $(a, f(a))$ .

False. It might not have either at  $x=a$ , like 

16) A human grows most rapidly when they are half of their mature height.

True.

17) The derivative of a sum is the sum of the derivatives.

True.


18) The derivative of a product is the product of the derivatives.

False. The product rule is not  $(f(x)g(x))' = f'(x)g'(x)$ .

19) A graph which has x-values from -1 to 16 is a good representative plot of  $f(x) = \frac{x^5}{5} - 16x$ .

False. It must include all critical points. It must include  $x = -2$ .  $f'(x) = x^4 - 16 = 0$   
 $x = \pm 2$

20) If  $f'[-3] = 0$  and  $f''[-3] = -3$ , then  $f(x)$  has a maximum at the point  $(-3, f[-3])$ .

True.  (Second Deriv. Test)

21) Logistic growth is sometimes called "controlled" exponential growth.

True.

22) The polynomial  $1 + x + \frac{x^2}{2} + \frac{x^3}{6}$  can be used to approximate  $e^x$ .

True.

**Multiple Choice [2 pts each]. Choose the letter of the best answer.**

23) If you want to graph your height from your birth until now, it is best to use a B to model the data.

- A) exponential curve
- B) logistic curve**
- C) polynomial approximation
- D) line with positive slope

24) If  $f'(a)$  is not zero, and  $f''(a)$  is zero, then  $f(x)$  has E at the point  $(a, f(a))$ .

- A) a maximum
- B) a minimum
- C) either a max or a min
- D) an inflection point
- E) don't know**

}  $f''$  must change signs to have an inflection pt.

25) If  $f'(a) = 0$ , and  $f''(a) > 0$ , then  $f(x)$  has B at the point  $(a, f(a))$ .

- A) a maximum
- B) a minimum**
- C) either a max or a min
- D) an inflection point
- E) don't know

→ 

26) If  $f'(a) = 0$ , and  $f''(a) = 0$ , then  $f(x)$  has E at the point  $(a, f(a))$ .

- A) a maximum
- B) a minimum
- C) either a max or a min
- D) an inflection point
- E) don't know**

27) A is a function with  $f[0] = 0.2$  and its instantaneous percentage growth rate is a constant 40%.

- (A)  $0.2e^{0.4x}$       B)  $0.2e^{4x}$       C)  $0.4e^{0.2x}$       D)  $40e^{0.2x}$

28) A crystal has grown in such a way that all the linear dimensions have increased by 75%.

$100\% + 75\%$

The new surface area is B times the original surface area.

- A)  $(0.75)^3$       (B)  $(1.75)^2$       C) 2      D) 1.75

29) A crystal has grown in such a way that all the linear dimensions have increased by 125%.

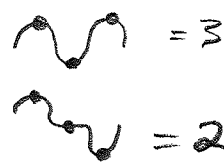
$(100 + 125)\%$

The new volume is B times the original volume.

- A)  $(1.25)^3$       (B)  $(2.25)^3$       C)  $(0.25)^3$       D) 3

30) If the equation  $f'[x] = 0$  has exactly 3 solutions, then we know B.

- A)  $f[x]$  has at least 3 crests or dips      (B)  $f[x]$  has at most 3 crests or dips  
C)  $f[x]$  has exactly 3 crests or dips      D) nothing



31) Given any interval  $[a, b]$ , there is an  $x$ -value  $c$  between  $a$  and  $b$ , such that  $f[b] - f[a] = f'[c](b - a)$ .

This is called the C.

- A) Point-Slope Formula      B) Law of Averages      (C) Law of the Mean      D) Race Track Principle

15+2  
□ Answer.

32) [2 pts] Fill in the blanks to make a correct version of the race track principle:

If  $f[b] = g[b]$ , and  $f'[x] \geq g'[x]$  for  $x < b$ , then  $f(x) \leq g(x)$  for  $x < b$ .

33) [2 pts] Write a sentence or two explaining a version (any version) of the race track principle, in terms of horses racing.

If two horses finish at the same place, and one horse always ran faster than the other, then the slower horse was ahead before they met.

34) [2 pts] A good representative plot of  $f[x] = x^3 - x$  should include what two values of  $x$ ? (show your work)

$$f'(x) = 3x^2 - 1 = 0$$

$$x^2 = \pm \frac{1}{3}$$

$$x = \pm \sqrt{\frac{1}{3}}$$

$$\left( \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right)$$

35) [2 pts] You know that  $y'[x] = 4y[x]$ , and that  $y[x]$  is positive for all  $x$ 's. What does the graph of  $\text{Log}[y[x]]$  look like? (Explain, don't draw.)

↓  
this means  $y(x) = ae^{4x}$

$$\begin{aligned} \text{Log}(y(x)) &= \text{Log}(a) + \text{Log}(e^{4x}) \\ &= \text{Log}(a) + 4x. \end{aligned}$$

It looks like  
a line with slope = 4

36) [2 pts] Given  $y'[x] = 0.7y[x] \left(1 - \frac{y[x]}{50}\right)$  with  $y[0] = 1$ , can  $y[x]$  ever go above  $y = 70$ ? Explain.

No.  $y(x)$  is a logistic curve with horizontal asymptote (max) at  $y = 50$ . It can never go above  $y = 50$ .

37) [2 pts] The quantity  $100 \frac{f'[x]}{f[x]}$  is called the Inst. Percentage Growth Rate of  $f[x]$ .

The quantity  $100 \left(\frac{f[x+1]}{f[x]} - 1\right)$  is called the (Average) Percentage Growth Rate of  $f[x]$ .

38) [3 pts] Rank these six functions in order of dominance (largest first and smallest last), as  $x$  approaches infinity:

$e^{0.2x}$   $\swarrow x'$   $\text{Log}[x]$   $\sqrt{x}$   $x \text{Log}[x]$   $e^{x^{0.2}}$   $\swarrow x^{1/5}$   $2x$   $\text{Log}[x] > 2$

---

$e^{0.2x}$   $e^{x^{0.2}}$   $x \text{Log}[x]$   $2x$   $\sqrt{x}$   $\text{Log}[x]$

Bonus) [2 pts] Rank these four functions in order of dominance as  $x$  approaches infinity (largest to smallest), and explain how you ranked  $\text{Log}[\text{Log}[x]]$ .

$x^2 = e^{2 \text{Log}[x]}$   $\text{Log}[x^2] = 2 \text{Log}[x]$   $\text{Log}[\text{Log}[x]]$   $x$

---

$x^2$   $x$   $2 \text{Log}[x]$   $\text{Log}[\text{Log}[x]]$

Since  $x > \text{Log}[x]$ ,  $\text{Log}(x) > \text{Log}(\text{Log}[x])$

4  Limits [2 pts each]. What is the limiting value?

Show some steps or explain your answer.

39)  $\lim_{x \rightarrow \infty} \frac{x^2}{x \text{Log}[x]} = \frac{x}{\text{Log}[x]} \rightarrow \frac{1x}{0x} \rightarrow \infty$  40)  $\lim_{x \rightarrow \infty} \frac{\text{Log}[x]}{\sqrt{x}} \rightarrow \frac{0\sqrt{x}}{1\sqrt{x}} = 0$

any positive power of  $x$  dominates  $\text{Log}[x]$

9  Matching.

41) [5 pts] Match the differentiation rule with its name. Write the correct Letter in the blank.

- B  $(x^r)' = r x^{r-1}$
- D  $(f[x] + g[x])' = f'[x] + g'[x]$
- F  $(f[g[x]])' = f'[g[x]] g'[x]$
- G  $(c f[x])' = c f'[x]$
- C  $(f[x] g[x])' = f'[x] g[x] + f[x] g'[x]$ .

- A. Exponential Rule
- ~~B~~. Power Rule
- ~~C~~. Product Rule
- ~~D~~. Sum Rule
- E. Quotient Rule
- ~~F~~. Chain Rule
- ~~G~~. Constant-Multiple Rule

42) [4 pts] Match the differential equation on the left with its solution  $y[x]$  on the right. Write the correct Letter in the blank.

C  $y'[x] = \frac{ry[x]}{x}$  Power

B  $y'[x] = ry[x]$  EXP

A  $y'[x] = ry[x] \left(1 - \frac{y[x]}{a}\right)$  Logistic

D  $y'[x] = r$  Linear

~~A.~~  $\frac{ace^{rx}}{a-c+ce^{rx}}$

~~B.~~  $ae^{rx}$

~~C.~~  $ax^r$

~~D.~~  $rx + a$

E.  $\sin[x]$

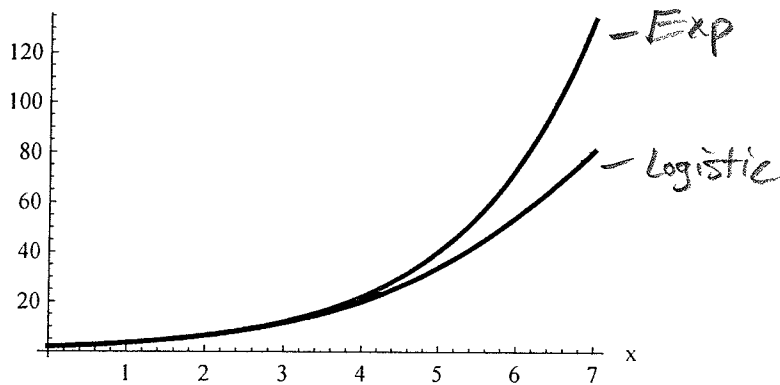
8+5  
□ Answer.

43) [4 pts] Use the race track principle to establish the trig identity  $\tan[-x] = -\tan[x]$ .

First, rewrite it as:  $\frac{\sin[-x]}{\cos[-x]} = -\frac{\sin[x]}{\cos[x]}$ .

Cancelled

44) [2 pts] Two functions are graphed on the plot below. They start at the same point and run together for a while before one grows higher than the other. Identify each type of function; choose from the following: exponential, power, linear, logistic, polynomial, euler faker.



Top = Exponential

Bottom = Logistic

