Math 220-X8 Test 3 Name____________________

☐ Calculate the Indefinite Integral [2 pt each].

1) $\int 7\, e^{x}\, dx$

2) $\int x^{-3}\, dx$

3) $\int (3 - x)\, dx$

4) $\int \left(\text{Cos}[x] + \frac{1}{x}\right) dx$

5) $\int \frac{\text{Cos}[e^{x}]\, e^{x}}{\text{Sin}[e^{x}]}\, dx$

☐ Calculate the Definite Integral [3 pts each]. Show some steps.

6) $\int_0^1 e^{x}\, dx$

7) $\int_1^2 (x^{-2} + 3x)\, dx$

8) $\int_0^2 \left(\frac{1}{x^4} - 3x + 1\right) dx$

9) $\int_0^{\pi/4} \frac{\text{Sin}[x]}{\text{Cos}[x]}\, dx$

10) $\int_0^2 (x^3 + 3x - 1)\, dx$

☐ True False False [2 pts each]. *If False, you must explain why.*

11) If f[x] is a function with even symmetry, then $\int_{-2}^{2} f[x]\, dx = 0$.

12) If f[x] is a function with odd symmetry, then $\int_{-6}^{6} f[x]\, dx = 0$.

13) If f[x] is a function with odd symmetry, then $\int_{-1}^{1} f[x]\, dx = 2 \int_{0}^{1} f[x]\, dx$.

14) $\int_{0}^{\pi} \mathrm{Sin}[x]\, dx = -\int_{\pi}^{2\pi} \mathrm{Sin}[x]\, dx$.

15) $\int_{a}^{b} f[x]\, dx$ is a number, but $\int f[x]\, dx$ is a function.

16) If f[x] > 0 for all x in the interval [a, b], then we know that $\int_{a}^{b} f[x]\, dx > 0$.

17) $\int_{0}^{a} \frac{1}{x}\, dx = \mathrm{Log}[a] - \mathrm{Log}[0]$.

18) $\int_{a}^{b} x^3\, dx = \frac{x^4}{4} + C$.

19) $\int_{a}^{b} (f[x] + g[x])\, dx = \int_{a}^{b} f[x]\, dx + \int_{a}^{b} g[x]\, dx$.

20) $\int_{a}^{b} f[x]\, g[x]\, dx = \left(\int_{a}^{b} f[x]\, dx\right)\left(\int_{a}^{b} g[x]\, dx\right)$.

21) $\left|\int_{-1}^{1} x\, dx\right| = \int_{-1}^{1} |x|\, dx$.

□ Multiple Choice [3 pts each]. Choose the letter of the best answer.

22) Consider the function y[x] which satisfies the differential equation $y'[x] = y[x]\,(2 + x^2 - y[x])$ and y[x] > 0 for all x. We know that y[x] will be decreasing during those intervals which make ______ true.

A) $y[x] > 2 + x^2$ B) $y[x] > x^2$ C) $y[x] < 0$ D) $y[x] < 2 + x^2$

23) Consider the same function y[x] as the previous problem. If y[x] has a minimum at x = a, then we know that ______ must be true.

A) $y[a] = 2 + a^2$ B) $y[a] = 0$ C) $y[a] = 0$ or $y[a] = 2 + a^2$

D) $2 + a^2 = 0$

24) The usual predator-prey model involves the simultaneous differential equations

$prey'[t] = a\, prey[t] - b\, prey[t]\, pred[t]$,

$pred'[t] = -c\, pred[t] + d\, pred[t]\, prey[t]$,

where a, b, c, and d are given positive constants. Which one of these statements is true? _______

A) When $prey[t] < \frac{c}{d}$, pred[t] is going up. B) When $pred[t] < \frac{a}{b}$, prey[t] is going up.

C) When $prey[t] = \frac{c}{d}$, pred[t] has a max. D) When pred[t] = 0 , prey[t] is going down.

25) Using the same predator-prey model as above, which of these statements is true? _______

A) If pred[t] = 0, then prey[t] won't change. B) If pred[t] = prey[t], then t = 0 .

C) If $pred[t] = \frac{a}{b}$, then prey[t] is neither increasing nor decreasing.

D) If a = b, then prey[t] is neither increasing nor decreasing.

26) The SIR infection model dealing with disease in a closed population (total population is constant) uses the three functions Sus[t], Inf[t], and Recov[t], which satisfy these simultaneous differential equations (a and b are positive constants):

$Sus'[t] = -a\, Sus[t]\, Inf[t]$,

$Inf'[t] = a\, Sus[t]\, Inf[t] - b\, Inf[t]$,

$Recov'[t] = b\, Inf[t]$.

After someone becomes infected and recovers from this disease, they become immune. Which of the following statements is FALSE? _____

A) If Sus[0] = 0, then there will be no epidemic. B) If Inf[0] = 0, then there will be no epidemic.

C) If Inf[t] and Sus[t] are positive, then Sus[t] is decreasing.

D) If $Sus[t] > \frac{b}{a}$, then Inf[t] is decreasing.

27) Using the same predator-prey model as above, which of these statements is FALSE? _______

A) As Inf[t] increases, Sus[t] decreases. B) As Inf[t] decreases, Recov[t] increases.

C) If $Sus[t] = \frac{b}{a}$, then Sus[t] has its maximum. D) If $Sus[t] = \frac{b}{a}$, then Inf[t] has its maximum.

28) If $f[t] = \int_0^t 4\, e^x\, Sin[x^3]\, dx$, then f'[t] = _____ .

A) $4\, e^t\, Sin[t^3]$ B) $4\, e^t\, Cos[t^3]\,(3\, t^2)$ C) $4\, e^t \int_0^t Sin[x^3]\, dx$ D) $\int_0^t 4\, e^x\, Cos[x^3]\,(3\, x^2)\, dx$

29) If $f[t] = \int_0^t Sin[x]\, dx$, then $f[\frac{\pi}{2}]$ equals _____ .

A) The area under $-Cos[x]$ from 0 to $\frac{\pi}{2}$. B) The area under Sin[x] from 0 to $\frac{\pi}{2}$.

C) $Sin[\frac{\pi}{2}]$ D) $Tan[\frac{\pi}{2}]$

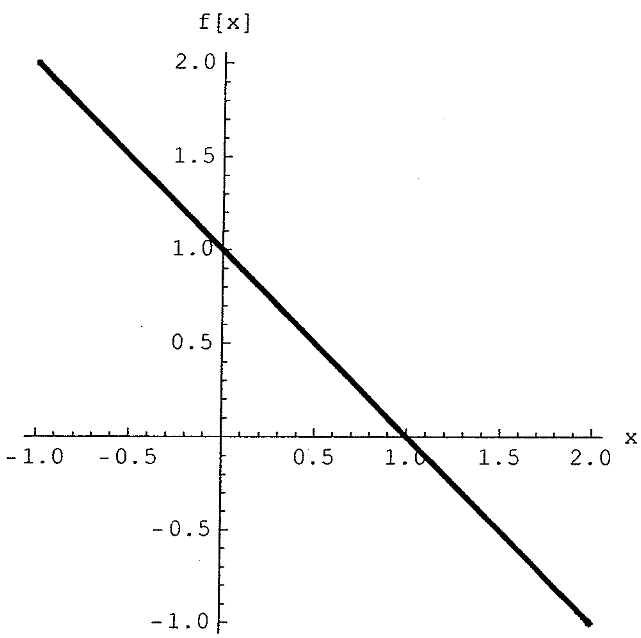
Various [2 pts each]. As always, show your work.

30) Look at the differential equation $y'[x] = y[x]\left(4 + \text{Sin}[x]^2 - y[x]\right)$ with $y[0] = 7$.
How do you know before you do any plotting that as x advances from 0, the plot of the solution y[x] initially must go down?

31) The acceleration of a falling object is $-32\ \frac{\text{ft}}{\text{s}^2}$. How far has a raindrop fallen 10 seconds after it leaves the cloud?

32) You have a box attached to a spring which is anchored to the ceiling. You hold the box up to the ceiling and let it go; it will fall down then spring back up toward the ceiling. The acceleration of the box is modeled by $a[t] = -2\,\text{Cos}[t]$. How much time has passed when the box arrives back up at the ceiling? (Note: velocity = 0 when this happens.)

33) Below is the graph of a function f[x] over the interval [−1, 2]. Use any method to calculate $\int_{-1}^{2} f[x]\,dx$.



34) Explain why sometimes Neptune is farther away from the Sun than Pluto is.

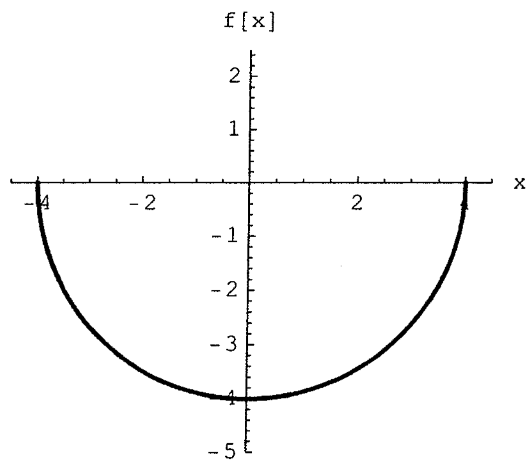
35) The logistic predator-prey model has the simultaneous differential equations

$$\text{prey}'[t] = a\,\text{prey}[t]\left(1 - \frac{\text{prey}[t]}{v}\right) - b\,\text{prey}[t]\,\text{pred}[t],$$

$$\text{pred}'[t] = -c\,\text{pred}[t] + d\,\text{pred}[t]\,\text{prey}[t]\,.$$

Explain why this model is more realistic than the simple predator-prey model (shown in question #24).

36-37) When you plot $f[x] = -\sqrt{16 - x^2}$ for $-4 \le x \le 4$, you get the bottom of the circle of radius 4 centered at {0, 0}:



~~That's the bottom half of a circle of radius 2.~~ Use this fact to calculate these two integrals:

36) [2 pts] $\int_{-4}^{4} f[x]\,dx$

37) [2 pts] $\int_{0}^{4} f[x]\,dx$.

□ Matching [2 pts each].

Match the parametrization with its plot. Write the correct Letter in the blank.

38) ___ $x[t] = \text{Cos}[t], \quad y[t] = 3\,\text{Sin}[t], \quad 0 \le t \le \pi$

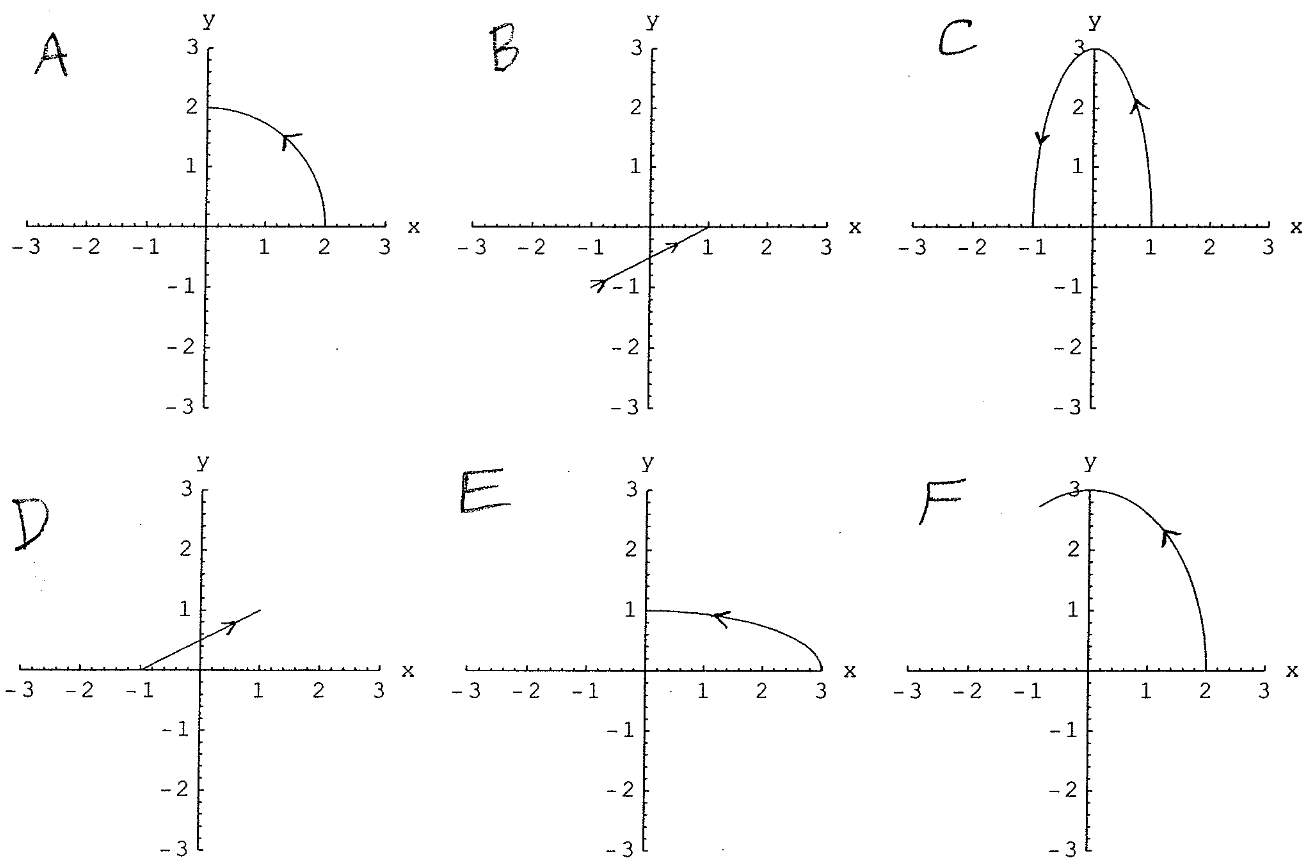
39) ___ $x[t] = 2t - 1, \quad y[t] = t, \quad 0 \le t \le 1$

40) ___ $x[t] = 2\,\text{Cos}[t], \quad y[t] = 2\,\text{Sin}[t], \quad 0 \le t \le \frac{\pi}{2}$

41) ___ $x[t] = 3\,\text{Cos}[t], \quad y[t] = \text{Sin}[t], \quad 0 \le t \le \frac{\pi}{2}$

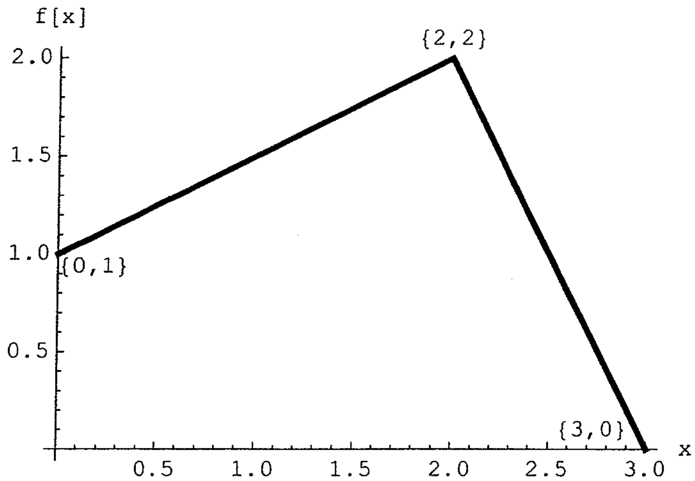
42) ___ $x[t] = 2\,\text{Cos}[2t], \quad y[t] = 3\,\text{Sin}[2t], \quad 0 \le t \le 1$

43) ___ $x[t] = 2t - 1, \quad y[t] = t - 1, \quad 0 \le t \le 1$



Bonus

B1) [2 pts] The graph of a function f[x] consists of two straight line segements. The first segment connects {0, 1} and {2, 2} and the second segment connects {2, 2} and {3, 0}. Here's the plot:



Measure some areas to come up with a calculation of

$\int_0^3 f[x]\, dx$.

B2) [3 pts]

- Calculate $\frac{d}{dx}(x\, \mathrm{Log}[x] - x)$.

- Use that information to calculate $\int_1^{e^5} \mathrm{Log}[x]\, dx$.