# NONSTANDARD ANALYSIS

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| MWF at 12:00 | Advanced Topics in Logic | Ward Henson |
| 343 Altgeld | Math 414 | Spring 2003 |

Nonstandard Analysis (NSA) is a framework for systematically applying some of the basic ideas of model theory to all areas of mathematics. It is especially effective in analysis, geometry, topology, and related areas of mathematics where the concept of *limit* is central.

Forty years ago, the logician Abraham Robinson observed that the construction of *nonstandard extensions* could provide a rigorous foundation for the use of infinitesimals in basic analysis.[1] Since then, applications of this set of ideas have spread through all of mathematics, greatly extending Robinson's original use of infinitely small and infinitely large numbers, and NSA has become an active branch of research in its own right.

In order to reach advanced applications of NSA in this course, we will assume a knowledge of first-order logic extending at least through the compactness theorem. Students should be able to formulate mathematical statements within first-order logic and should have some experience with nonstandard models. We will also use some ideas (such as the construction of *saturated models*) from the beginning parts of model theory.

After developing the basic framework of NSA we will give a substantial indication of how NSA is applied within three areas of advanced mathematics:
• probability and stochastic analysis (based on the Loeb measure construction);
• geometry and functional analysis (based on the nonstandard hull construction);
• asymptotics for ordinary differential equations.

*Prerequisites:* A knowledge of first-order logic through the compactness theorem; what is covered in the first half of Math 410 at UIUC or in a good undergraduate course in logic will be sufficient.

*References:* There will be no text for the course, and a set of class notes will be prepared during the semester by the instructor and the students. Two books that cover basic material such as will be taught in this course (but which are at a slightly more elementary level) are:
• *Lectures on the Hyperreals; an Introduction to Nonstandard Analysis*, by Robert Goldblatt, Springer-Verlag, Graduate Texts in Mathematics 188, 1998.
• *An Introduction to Nonstandard Real Analysis*, by Hurd and Loeb, Adademic Press, 1985; reprinted 1998.

The following book contains a collection of articles covering basic NSA as well as advanced material from many of the areas of mathematics to which NSA is being applied:
• *Nonstandard Analysis: Theory and Applications*, eds. Arkeryd, Cutland, and Henson, NATO ASI Series C Vol. 493, Kluwer, 1997.

*Grading:* This is primarily a lecture course. Students will help the instructor produce a set of class notes for the course, during the semester. There will be regular assignments of homework problems, on the understanding that such work is essential to learning any area of mathematics. Students will be encouraged to give a lecture on a topic or project of their choice.

[1] A. Robinson, Non-standard analysis, *Proc. Royal Acad. Amsterdam Ser. A*, **64** (1961), 432–440.