Name:

Collaborator(s)[1]:

Math 213, Section F1, Prof. Hildebrand, Fall 2010
Graded HW Assignment 3, due Friday, 9/24/2010

Instructions

- **Use this sheet as cover sheet and staple it to the assignment.** Do the problems in order, and make sure that each problem is clearly labelled. Leave plenty of space for the problems. **The assignment is due in class on the above date; late homework, or homework dropped off in mailboxes, will not be accepted.** See the Course Information Sheet for the policy on "excused" homework.
- **Write-up:** Solutions, rather than answers, are required. An answer alone will not earn credit. The solutions must be written up in a clear, logical manner, using correct mathematical terminology and notation, and any key steps explained. **See below for any specific instructions on the write-up.**
- **Getting Help:** Open House hours are Wednesdays/Thursdays, 5 pm - 6 pm, in 159 Altgeld. The Open House is intended as informal get-together and office hour for students in my casses. Take advantage of this opportunity!
- **Group work policy:** Work on the problems with another student or in a small group is fine and, indeed, encouraged, **provided** (i) you write up solutions yourself, using your own words, and (ii) you indicate the names of the student(s) you worked with on the cover sheet.

Problems (from Rosen, 6th Edition, Sections 5.1 and 5.2)

1. 5.1: 8
2. 5.1: 14
3. 5.1: 16
4. 5.1: 17
5. 5.1: 23
6. 5.1: 24
7. 5.1: 28
8. 5.1: 30 (all parts)
9. 5.1: 34
10. 5.1: 38
11. 5.1: 39
12. 5.1: 43
13. 5.2: 2*
14. 5.2: 6* (cf. 5)
15. 5.2: 14* (cf. 13)
16. 5.2: 29
17. 5.2: 40

See back of the page for instructions on the write-up.

[1] If you worked with another student or in a small group on this assignment, list the names of all students involved.

Instructions

- **Write-up for problems from 5.1:** Most problems from 5.1 have a numerical answer (such as $15,600$ or $63,273,600$). These numerical answers are the result of multiplying out "raw" counts like $26 \cdot 25 \cdot 24$ or $26^6 + 26^6 - 26^4$. Leave your answers in the latter, "raw", form, rather than getting a numerical answer. **For grading purposes, a numerical answer is irrelevant, and a numerical answer alone will not earn credit. You must show how you arrived at your answer, for example, using a notation such as** $\underbrace{26 \cdot 26}_{\text{2 letters}} \cdot \underbrace{10 \cdot 10 \cdot 10}_{\text{3 digits}}$**.**

- **Write up for problems from Section 5.2:** All of these problems are pigeonhole problems, and should be solved by applying the pigeonhole principle with **appropriately defined** "pigeonholes" and "pigeons". "Appropriately defined" is the key here, the definition is an essential part of a solution, and it must be done precisely. Thus, for each problem, do the following (in addition to stating an overall conclusion that answers the question):

  - Clearly state what you define as the "pigeonholes", and their number (the "k" in PHP).
  - Clearly state what you define as the "pigeon", and their number (the "N" in PHP).

  By doing so, you show that you understood how to model the given problem by a pigeon/pigeonhole situation. A correct identification of pigeons and pigeonholes is essential for credit on these problems.

  See the class examples for models.

Tips

About these problems. The problems in Section 5.1 represent the simplest type of combinatorial problems, those that require no special techniques and formulas. Most are on the easy side and should take little time, but you need to carefully read each question and analyze the given situation in order to come up with the correct counting procedure. Section 5.2 covers a special counting technique, the pigeonhole principle.

Getting proficient with combinatorial problems requires lots of practice—hence the relatively large number of problems in this assignment.

Here is some advice on approaching this assignment.

- **Review your class notes and study the examples in the text:** If you have not done so, read Section 5.1 (you can skip the final part, on tree diagrams), and study the following Examples: 1-8, 10, 11, 12, 15-18. *Do this before starting to work on the problems.*

- **Common strategies:** For many problems the slot counting method discussed in class (where you imagine filling slots left to right and count the number of choices you have for each slot) works like a charm. For some problems a simple "inclusion-exclusion" argument (adding individual counts and then subtracting the overlap) does the trick. Finally, for some problems the complement trick (count the complement, then subtract from the total) is essential.

- **"Twin" problems:** Several of the assigned problems from 5.2 have odd-numbered "twins", which are problems of exactly the same type, except for different numbers. These twin problems, indicated in parentheses on the problem sheet, have solutions in the back of the book. If you get stuck on an assigned problem, look up the corresponding twin. (The twin problems are not to be turned in.)