MATH 242 X **Exam 1** **Fall 2000**

1. *(15) [Quickies; 5 points per question]* *Given the points* $P(120, 130, 242)$ *and* $Q(242, 130, 120)$, *compute the following quantities:*
(i) *The vector* $\overrightarrow{PQ}$.
(ii) *The position vector* $\overrightarrow{0M}$ *of the midpoint* M *of the line segment* PQ.
(iii) *Parametric equations of the line passing through the points* P *and* Q.

2. *(25) [Quickies; 5 points per question] Given the vectors* $\overrightarrow{a} = \langle 2, 1, 0\rangle$ *and* $\overrightarrow{b} = \langle 1, 1, -1\rangle$, *compute the following quantities. (The answers should be in numerical form, such as* $\langle 2, 4, 2\rangle$ *or* $1/\sqrt{2}$.*)*
(i) *(5) The* **scalar** *projection of* $\overrightarrow{b}$ *onto* $\overrightarrow{a}$.
(ii) *(5) The* **vector** *projection of* $\overrightarrow{a}$ *onto* $\overrightarrow{b}$. *(Note that the order of* $\overrightarrow{a}$ *and* $\overrightarrow{b}$ *here is different from that in part (i)!)*
(iii) *(5) The* **cosine** *of the angle between* $\overrightarrow{a}$ *and* $\overrightarrow{b}$.
(iv) *(5) The area of the parallelogram spanned by* $\overrightarrow{a}$ *and* $\overrightarrow{b}$.
(v) *(5)* **Two** *unit vectors that are orthogonal to both* $\overrightarrow{a}$ *and* $\overrightarrow{b}$.

3. *(10) A particle moves along a circle of radius* $R = 3$ *at a speed* $v(t) = t^2$ *at time* t. *Compute the following quantities.*
(i) *(5) The tangential component* a_T *of the acceleration at time* $t = 3$.
(ii) *(5) The normal component* a_N *of the acceleration at time* $t = 3$.

Note: There exist two sets of formulas for a_T and a_N, one involving the speed v and the curvature κ, and the other involving $\overrightarrow{r}\,'$ and $\overrightarrow{r}\,''$. For this problem the first set of formulas had to be used, since the problem involved v and κ, but not $\overrightarrow{r}\,'$ and $\overrightarrow{r}\,''$. (The curvature κ wasn't directly given, but could be obtained from the radius R via the formula $\kappa = 1/R$.)

4. *A force* $\overrightarrow{F} = \langle 0, 5, 1\rangle$ *acts on a particle of mass* $m = 1/2$, *which at time* $t = 0$ *is located at the origin and has velocity* $\overrightarrow{v}(0) = \langle 1, 0, 0\rangle$.
(i) *(5) Determine the acceleration* $\overrightarrow{a}(t)$ *and velocity* $\overrightarrow{v}(t)$ *of the particle at any time* t.
(ii) *(5) Determine the position of the particle at time* $t = 2$.

5. *(10) Let* P *denote the plane determined by the points* $A(1, 0, 0)$ *and* $B(2, 0, -1)$, *and* $C(1, 4, 3)$.
(i) *(5) Find a linear equation for* P.
(ii) *(5) Find the distance from* P *to the origin.*

6. *(20) Given the space curve* $\overrightarrow{r}(t) = \langle 3\sin t, -3\cos t, 4t\rangle$, *compute the following quantities:*
(i) *(5) The unit tangent vector* $\overrightarrow{T}$ *at* $t = \pi/2$.
(ii) *(5) The unit normal vector* $\overrightarrow{N}$ *at* $t = \pi/2$.
(iii) *(5) The binormal vector* $\overrightarrow{B}$ *at* $t = \pi/2$.
(iv) *(5) The linear equation of the osculating plane at* $t = \pi/2$.

7. *(10) Let* $\overrightarrow{r}(t)$ *denote the position of a moving particle at time* t, *and let* $\overrightarrow{v}(t)$ *be its velocity, and* $v(t)$ *its speed at time* t. *Assume that the angle between the vectors* $\overrightarrow{r}(t)$ *and* $\overrightarrow{v}(t)$ *is always* $\pi/4$ *(i.e., 45 degrees). Find a formula for* $\frac{d}{dt}|\overrightarrow{r}(t)|$, **simplifying as much as possible.** *(In particular, your formula should not involve* $\overrightarrow{r}(t)$.*) Show all work, and justify every step in your argument! (Hint: First find a general formula for* $\frac{d}{dt}|\overrightarrow{r}(t)|$ *in terms of* $\overrightarrow{r}(t)$ *and* $\overrightarrow{r}\,'(t)$, *then simplify this formula.)*