Name:

Math 241, Section F1H, Fall 2007
Graded HW Assignment 10, Due 11/5/2007

1. **Finding a potential function for a given vector field $\vec{F}$.** Use the *systematic* approach I used in class (via "partial integration", e.g., as in Example 5 in 14.3), rather than what the book calls the "inspection" method (which really means just plain guessing). Once you have come up with an f, you can check your answer by computing the partial derivatives f_x and f_y, and checking whether those are equal to the given functions P and Q.

2. **Finding a potential function for a three-dimensional vector fields.** This is needed in 14.3:28. The basic procedure for finding a potential is the same as in the 2-dimensional case, but the "integration constants" are now functions of two (instead of one) variable: The first step is to integrate P with respect to x and set the result equal to $f(x, y, z)$. This gives $f(x, y, z) =$ something $+ g(y, z)$, where $g(y, z)$ represents the "integration constant". Next, differentiate $f(x, y, z)$ with respect to y (i.e., computing f_y) and set the result equal to Q to obtain an equation of the form (1) $g_y(y, z) =$ something. Similarly, computing f_z one obtains an equation of the form (2) $g_z(y, z) =$ something, where "something" stands for some function of y and z. Now the equations (1)–(2) combined represent a 2-dimensional "find the potential" problem, and can be solved as such to find $g(y, z)$, and hence $f(x, y, z)$.

3. **Problem 14.3:30** This is the example mentioned in class in which independence of path does in general not hold, due to a "hole" in the domain at $(0, 0)$. Just compute the integral over each of the two paths *explicitly* (i.e., by parametrizing each of the two paths and calculating the line integral using this parametrization). You can ignore the two questions at the end since these were addressed in class.

4. **Problem 14.3:36** To compute the line integral representing the work, find a potential for the given field $\vec{F}$. (Hint: This came up in a problem in 14.1.)

5. **Problem 14.4:27** This is a calculation involving the ∇ operator, unrelated to Green's Theorem, of the type that came up in 14.1 (see 14.1:33). Note that ∇^2 is to be interpreted as the dot product $\nabla \cdot \nabla$, and evaluate the left side explicitly, using the definition $\nabla = \langle \partial/\partial x, \partial/\partial y \rangle$ and the product rule for partial derivatives. After a couple of intermediate steps, you should end up with the expression on the right of the equation.

Problems

1. 14.3:6
2. 14.3:8
3. 14.3:16
4. 14.3:23
5. 14.3:28
6. 14.3:30
7. 14.3:36
8. 14.4:3
9. 14.4:4 (Answer: 1/5)
10. 14.4:13
11. 14.4:17
12. 14.4:21
13. 14.4:22 (Answer: $243\pi/2$)
14. 14.4:27
15. 14.4:29