Name:

Collaborator(s)[1]:

Math 347, Section E1H, Prof. Hildebrand, Spring 2012

HW Assignment 1, due Friday, 1/27/2012

Instructions:

- The assignment is due in class at the above due date. **Use this sheet as cover sheet and staple it to the assignment.** Do not turn in loose sheets. Do the problems in order, and make sure that each problem is clearly labelled. Write legibly, and allow plenty of space for each problem (half a page or more is needed for many problems). **Group work policy:** Work on the problems with another student or in a small group is fine and, indeed, encouraged, **provided** (i) you write up solutions yourself, using your own words, and (ii) you indicate the names of the student(s) you worked with on the cover sheet.
- **Getting help:** The best place is the Open House, Sundays beginning at around 3 pm, Tuesdays and Thursdays beginning at 5:15 pm, all in 159 Altgeld. I'll stay as long as necessary. The Open House is a good place to get to know other students and find someone compatible to work with. Also, I'll be happy to look over draft solutions, and give hints if you get stuck.
- **Write-up:** All problems require a properly written up solution/proof, not just an answer, or a disconnected bunch of equations. Grading will be based on the quality of this write-up; for full credit it is essential that you present your solution/proof in a clear, logical manner, using complete sentences, proper terminology and correct mathematical notation. **Note that words like "determine", "show", "obtain" in an exercise have essentially the same meaning as "prove" (cf. the comment at the beginning of the exercise section on p. 20).**

HW 1 Problems (from D'Angelo/West, Chapter 1)

1. #1.22 (Water/Wine Puzzle – see text for precise statement)
2. #1.27. Let $S = \{x \in \mathbb{R} : |x/(x+1)| \leq 1\}$ and $T = \{x \in \mathbb{R} : x \geq -1/2\}$. Show that $S = T$.
3. #1.32. Let $S = \{x \in \mathbb{R} : x^2 - 2x - 3 < 0\}$ and $T = \{x \in \mathbb{R} : -1 < x < 3\}$. Show that $S = T$.
4. #1.50. Let C and D denote subsets of the domain of f. Prove that $f(C \cap D) \subseteq f(C) \cap f(D)$.

   In the remaining problems, A, B, C denote arbitrary sets.

5. Show that $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.
6. Show that $(A - B) \cup (A - C) = A - (B \cap C)$.
7. Show that $(A \cup B) - C \subseteq (A - (B \cup C)) \cup (B - (A \cap C))$. Also give an example showing that equality does not necessarily hold.
8. Show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

*** Turn page for instructions and hints ***

[1] If you worked with another student or in a small group on this assignment, list the names of all students involved.

HW 1 Comments and Hints

- **Problem 1.22:** This is a fun puzzle with a surprising answer (try it on a friend!). Getting this answer is a matter of high school level algebra, and not the main point of the problem. Rather, the problem is intended as practice for writing up a mathematical argument in a clear, logical manner. Thus, focus on this write-up, making sure to use complete sentences, with full explanations. (Note: The intended solution is a rather simple exercise in high school level algebra. However, there is another, completely different solution that is very slick and elegant, requires no calculations, and proves a much stronger statement than that given in the problem. You'll know it when you get it.)

- **Problems 1.27 and 1.32:** These problems ask to prove equalities between sets of *real numbers*, i.e., sets of the type $\{x \in \mathbb{R} : \dots\}$, where the dots represent some constraint on x. Proceed as in Example 1.13. In particular, split the proof into two parts, $S \subseteq T$ and $T \subseteq S$. To prove $S \subseteq T$, show that any real number x satisfying the conndition in the definition of S also satisfies the condition in the definition of T; to prove $T \subseteq S$, show the reverse implication. For these proofs, you can make use of the standard algebraic properties of real numbers and the properties of inequalities and absolute values. (Hint: A useful trick with such problems is to complete the square.) (Note that the statement of 1.27 in the text is a bit different than the one given here, but amounts to the same thing. I have rephrased the problem in a manner analogous to 1.32 and Example 1.13.)

- **Problem 1.50:** This is an exercise in proving general set-theoretic relations involving sets of the form $f(S) = \{f(s) : s \in S\}$, where f is a function from A to B and S is a subset of A. Although this problem involves the concept of a function (which comes up in Chapter 1, but which we'll cover later in class), the only thing you need to know about functions is the above definition of $f(S)$. Hint: This definition can also be written as $\{b \in B :$ there exists $a \in S$ such that $f(a) = b\}$.

- **Problems 5–8:** These problems ask you to prove identities or subset relations for arbitrary sets. In each case, give a formal, step-by-step proof, following the model given in class (i.e., a proof of the form "Let $x \in \dots$" [sequence of logical steps] "Therefore $x \in \dots$"). The proof should only use the definitions of set operations (e.g., $x \in A - C$ means "$x \in A$ and $x \notin C$"). **Be sure to justify each step, and use appropriate connecting words ("then", "therefore", etc.)**.

  Note on Venn diagrams: Drawing a Venn diagram *on scratch paper* can be very useful in picturing what is going on, and in coming up with counterexamples if needed. However, a Venn diagram does not constitute a proof, and your argument should not depend on Venn diagrams.