Name (PRINT):

Honor statement: I have not received assistance from others or consulted references other than the text for this assignment. **Signed:** ______________________

Math 347, Section E1H, Prof. Hildebrand, Spring 2012

Honors HW Assignment 1, due Monday, 2/13/2012

Instructions

- **Absolutely no groupwork, consulting with friends, classmates, roommates, no googling, etc..** You should do this assignment entirely on your own, as if it were a take-home exam. You can use the class notes and the text, but no other books, or online materials. By signing the honor statement above, you confirm that you played by these rules.
- **Use this sheet as cover sheet and staple it to the assignment.** Do not turn in loose sheets. Do the problems in order, and make sure that each problem is clearly labelled. Write legibly, and allow plenty of space for each problem; it is best to do each problem on a separate page.
- **Deadline:** For the honors assignments I am somewhat flexible with the deadlines. If you'd like an extension, just ask! If it's a reasonable request, I'll likely grant it.
- **About this assignment:** As announced on the Course Syllabus handed out at the beginning of the semester, in addition to the regular homework assignments, there will be periodic honors level assignments consisting of less routine problems. This is the first such assignment. All of the problems on this assignment are a bit out of the ordinary in one way or another. Some are not particularly difficult, but more laborintensive than a regular homework problem; others require some special insight, but don't take very long once you have the right idea. Yet others present surprising applications of techniques you learned in class.

  Some of the problems are from the text, some are from other sources, and some I made up myself. I tried very hard to pick problems that are both interesting and intellectually challenging and that are at least loosely related to what we have covered in class. I hope you'll enjoy these problems and the challenge they present. The problems vary in difficulty. While some should be doable by nearly every student in this class, others are intended to present a worthy challenge for the very best students in this class. Don't feel bad if there are problems you cannot see a way of doing!

  Since the honors assignments serve a different purpose than the regular homework assignments, the rules are different from, and stricter than, those for regular assignments. **In particular, for honors assignments group work is not permitted. You should work on these assignments entirely on your own and not discuss the problems with classmates and friends.** You can use the class notes and the text, but not other books or online sources. Do not try to locate a solution online by googling. This is against the rules, it would defeat the purpose of the assignment, and it would deprive you of the satisfaction and pride you get when you manage to solve a challenging problem on your own!

  I am happy to look at draft solutions and offer comments, but I cannot give away key ideas in solving a problem as this would defeat the purpose of the assignment. In most cases, if you are on the right track, you'll know it!

***** Problems on back of page *****

Honors HW 1 Problems

1. **[1.25] (Census taker problem)**
2. **[2.11] (Penny/dime/dollar puzzle)**
3. **[2.32] (Liars puzzle)**
4. **The sum/product mystery.** *Two integers in the range $\{2, 3, ..., 99\}$ are secretly drawn. The sum of the two integers is given to Dr. S, and the product is given to Dr. P. Dr. S and Dr. P, both of whom are excellent mathematicians, then engage in the following short dialogue:*

   Dr. P: I do not know the two numbers.

   Dr. S: I know that you didn't know the two numbers.

   Dr. P: Now I know the two numbers.

   Dr. S: Now I know the two numbers.

   What were the two numbers drawn?

   Remark: *If you have figured out the census taker puzzle (1.25 from the text), you should be able to do this one as well since the underlying idea is the same. Another problem based on this idea is 1.26 in the text, but the calculations needed for this problem are extremely tedious, and it would take hours to get through all the cases by hand. I like the sum/product version best since it illustrates this idea in its purest and starkest form, which makes the result all the more surprising.*

5. **Understanding $\epsilon - \delta$ statements.** *Each of the following statements is a "botched" version of a definition about a function f from $\mathbb{R}$ to $\mathbb{R}$. Determine,* in as simple language as possible, *what the statement really defines, i.e., determine the exact set of functions $f : \mathbb{R} \to \mathbb{R}$ that statisfy the statement. (This is a variation on Problem 2.27; see also Problem 2.24 from HW 2 and Problem 5 from the Logic Worksheet for problems of similar flavor. Examples of "simple language" descriptions might be "f is bounded on some finite interval", "f is a constant function", "f is any function such that $f(x) \to 0$ as $x \to \infty$", etc.)*

   (a) *For all $x \in \mathbb{R}$ and for all $\delta > 0$ there exists $\epsilon > 0$ such that $|x| < \delta$ implies $|f(x)| < \epsilon$.*

   (b) *There exists $\delta > 0$ such that for all $\epsilon > 0$ and for all $x \in \mathbb{R}$, $|x| < \delta$ implies $|f(x)| < \epsilon$.*

   (c) *For every $\epsilon > 0$ there exists $x \in \mathbb{R}$ such that for all $\delta > 0$, $|x| < \delta$ implies $|f(x)| < \epsilon$.*

6. **Almost (?) proof of a famous conjecture.** *An integer n that can be expressed as a sum of two prime numbers is a Goldbach number. For example, $4 = 2 + 2$, $6 = 3 + 3$, $8 = 3 + 5$, $10 = 3 + 7$, $12 = 5 + 7$, $14 = 3 + 11$, $16 = 3 + 13$ are all Goldbach numbers. One of the most famous unsolved problems in number theory is Goldbach's conjecture which states that every even integer greater than 2 is a Goldbach number. The above hand calculation shows that the conjecture is true up up to 16. These calculations have been extended, first by hand, and then using computers, and the current record is 10^{18}: That is, every even number greater than 2 and below 10^{18} has been checked and found to be a Goldbach number.*

   Despite this extensive numerical evidence, the Goldbach conjecture, which states that all even numbers greater *than 2 are Goldbach numbers, remains unresolved. Since there are infinitely many even numbers, it seems inconceivable to obtain the conjecture by computations alone. Yet, there is the following theorem, which in essence says that a* finite amount of computation *is enough to imply the full (infinite) Goldbach conjecture:*

   > *There exists an integer $N > 4$, such that if every even number n in the range $4 \leq n \leq N$ is a Goldbach number, then all even $n \geq 4$ are Goldbach numbers. In other words, there exists a bound N such that to prove the full Goldbach conjecture it suffices to check that it holds up to N.*

   Your task is to prove this statement. As mentioned above, the argument requires zero knowledge of number theory or primes, but a good dose of logic and some very clever thinking. There are no tricks involved here. The above is a perfectly valid mathematical statement, and there is an ironclad proof for it. As with all honors hw problems, you should arrive at the solution on your own. ***Don't ask friends or consult books or google (the problem is rather obscure and it's quite unlikely that solutions are floating on the internet). Also, if you have found a solution (you'll know it—but if you aren't sure, I'd be happy to take a look), keep it to yourself.***