Name:

Collaborator(s)[1]:

Math 347, Section E1H, Prof. Hildebrand, Spring 2012

HW Assignment 3, due Friday, 2/10/2012

- **Rules:** The usual: The assignment is due in class at the above due date. Use this sheet as cover sheet and staple it to the assignment. Do not turn in loose sheets. Do the problems in order, and make sure that each problem is clearly labelled. Write legibly, and allow plenty of space for each problem (half a page or more is needed for many problems. Group work is okay provided you write up solutions yourself, using your own words, and you indicate the name(s) of the student(s) you worked with on the cover sheet.
- **About this assignment:** This is the first of several problem sets on induction. The problems in this set are among the simplest induction problems. The proofs all fall into one of the three basic types of induction proofs given on the Induction Worksheet, and you can use the worksheet examples as models and templates..

  A key purpose of this assignment is to practice the proper write-up of induction proofs. Induction proofs (especially, routine type proofs like those in this assignment) provide an ideal setting in which to practice and perfect your proof writing skills. **You should take the write-up seriously and strive for a proof that is as close to perfect as possible in every respect: the logic, the mathematics, the notation, the visual presentation (e.g., display long chains of equations), and the English (e.g., use proper spelling, grammar, and punctuation).** Use the examples on the worksheet as models.

  In particular, an induction proof should include a precise statement of the proposition $P(n)$ to be proved, a base case, an induction step, and a conclusion. The proof of the induction step is the crux of the argument, and it must be given in full detail, with each of the steps justified (marginal notes like "by inductive hypothesis" or "by algebra" are okay).

HW 3 Problems

1. 3.15 *Prove by induction that, for all* $n \in \mathbb{N}$, $\sum_{i=1}^n (-1)^i i^2 = (-1)^n n(n+1)/2$.
2. 3.22 *Prove that, for any* $n \in \mathbb{N}$ *and any real numbers* $a_1, a_2, \dots, a_n$, $|\sum_{i=1}^n a_i| \le \sum_{i=1}^n |a_i|$. *(You may use the Triangle Inequality, which says that* $|x+y| \le |x| + |y|$ *for any real numbers* x *and* y. *The statement to be proved is a generalized version of this inequality.)*
3. 3.28 *Find and prove by induction a formula for* $\sum_{i=1}^n \frac{1}{i(i+1)}$, *where* $n \in \mathbb{N}$.
4. 3.29 *Find and prove by induction a formula for* $\sum_{i=1}^n (2i-1)$ *(i.e., the sum of the first* n *odd numbers), where* $n \in \mathbb{N}$.
5. 3.31 *Find and prove by induction a formula for* $\prod_{i=2}^n (1 - \frac{1}{i^2})$, *where* $n \in \mathbb{N}$ *and* $n \ge 2$.
6. 3.32 *Find and prove by induction a formula for* $\prod_{i=2}^n (1 - \frac{(-1)^i}{i})$, *where* $n \in \mathbb{N}$ *and* $n \ge 2$. *(Hint: Distinguish between even and odd values of* n.)
7. 3.43 *Suppose* $f : \mathbb{R} \to \mathbb{R}$ *satisfies*

   $$(*) \qquad f(xy) = xf(y) + yf(x)$$

   for all $x, y \in \mathbb{R}$. *Prove that* $f(1) = 0$ *and that* $f(u^n) = nu^{n-1} f(u)$ *for all* $n \in \mathbb{N}$ *and* $u \in \mathbb{R}$.
8. 3.49(b) *Determine the exact set of natural numbers* n *for which the inequality* $2^n \ge (n+1)^2$ *holds.*

[1] If you worked with another student or in a small group on this assignment, list the names of all students involved.