Logical statements: Summary of terminology and notations

- **Logical operations:** $\wedge$ ("and"), $\vee$ ("or"), $\neg$ (negation). You should know the truth tables for these operations, their set-theoretic analogs (see table on p. 34), and basic rules such as De Morgan's Law (see below). Note that "or" in mathematics is always non-exclusive; i.e., if P and Q are both true, then $P \vee Q$ is true.
- **Quantifiers:** $\exists$ ("there exists") $\forall$ ("for all"). Here are some English phrases that translate into quantifiers (cf. the table on p. 29).
  - "Every $x \in S$ satisfies ...": $(\forall x \in S) \dots$.
  - "For any $x \in S$, ...": $(\forall x \in S) \dots$.
  - "Given $x \in S$, we have ...": $(\forall x \in S) \dots$.
  - "If $x \in S$, then ...": $(\forall x \in S) \dots$.
  - "Whenever $x \in S$, then ...": $(\forall x \in S) \dots$.
  - "... holds whenever $x \in S$": $(\forall x \in S) \dots$.
  - "... holds for some $x \in S$": $(\exists x \in S) \dots$.
  - "For some $x \in S$ we have ...": $(\exists x \in S) \dots$.
  - "There is $x \in S$ we have ...": $(\exists x \in S) \dots$.
- **Conditionals:** "$P \Rightarrow Q$" ("if P, then Q", "P implies Q") is true if and only if either P is false, or P is true and Q is true. "$P \Rightarrow Q$" is logically equivalent to "$(\neg P) \vee Q$". and "$P \Leftrightarrow Q$" means that both "$P \Rightarrow Q$" and "$Q \Rightarrow P$", i.e., that P and Q are either both true or both false. Here are some English phrases and their translations into this notation. (See also the table on top of p. 33.)
  - "If P, then Q": $P \Rightarrow Q$
  - "P implies Q": $P \Rightarrow Q$
  - "P follows from Q": $Q \Rightarrow P$
  - "P is true whenever Q is true": $Q \Rightarrow P$
  - "P is true if Q is true": $Q \Rightarrow P$
  - "P is true only if Q is true": $P \Rightarrow Q$
  - "P is a sufficient condition for Q": $P \Rightarrow Q$
  - "P is a necessary condition for Q": $Q \Rightarrow P$
  - "P is equivalent to Q": $P \Leftrightarrow Q$
  - "P holds if and only if Q holds": $P \Leftrightarrow Q$
  - "P is a necessary and sufficient condition for Q": $P \Leftrightarrow Q$
- **Rules for negation:**
  - $\neg(P \wedge Q) \Leftrightarrow (\neg P) \vee (\neg Q)$ (De Morgan's Law, I)
  - $\neg(P \vee Q) \Leftrightarrow (\neg P) \wedge (\neg Q)$ (De Morgan's Law, II)
  - $\neg(\forall x \in S)(P(x)) \Leftrightarrow (\exists x \in S)(\neg P(x))$
  - $\neg(\exists x \in S)(P(x)) \Leftrightarrow (\forall x \in S)(\neg P(x))$
- **Further resources:** This material is covered in the D'Angelo/West text on pp. 27–34. A great source for additional examples and problems is Chapter 1 of the Rosen text "Discrete Mathematics" (on reserve in the Math Library).

The fact that all Mathematics is Symbolic Logic is one of the greatest discoveries of our age. —*Bertrand Russell*

Logic, like whiskey, loses its beneficial effect when taken in too large quantities. —*Lord Dunsany*