Set-theoretic notation and terminology: Practice problems

Solutions

1. Write the following sets using "set builder" notation:

   (a) The set of *real* solutions to $x^2 - 2x < 3$. $\boxed{\{x \in \mathbb{R} : x^2 - 2x < 3\}}$

   (b) The set of *integer solutions* to $x^2 - 2x < 3$. $\boxed{\{x \in \mathbb{Z} : x^2 - 2x < 3\}}$

   (c) The set of odd integers. $\boxed{\{2k + 1 : k \in \mathbb{Z}\}}$

   Alternatively, $\{n \in \mathbb{Z} : n = 2k + 1$ *for some* $k \in \mathbb{Z}\}$.

   (d) The set of odd natural numbers. $\boxed{\{2k - 1 : k \in \mathbb{N}\}.}$

   Note that we need $2k - 1$, *not* $2k + 1$, *so that the number 1 is counted. An alternative would be* $\{2k + 1 : k \in \mathbb{N} \cup \{0\}\}$.

   (e) The set $\{1, 4, 7, 10, \ldots\}$. $\boxed{\{2k + 1 : k \in \mathbb{Z}\}}$

   (f) The set of integers that can be written as sums of two integer squares. (The first few such integers are $0 = 0^2 + 0^2$, $1 = 0^2 + 1^2 = 1$, $2 = 1^2 + 1^2$, $4 = 0^2 + 2^2$.) $\boxed{\{(n, m) \in \mathbb{Z}^2 : n, m \in \mathbb{Z}\}}$

2. Given $A = \{1, 2, 3, 4\}$, $B = \{0, 1\}$, find the following sets. *Be sure to use correct set-theoretic notation.*

   (a) $A - B$ $\boxed{\{2, 3, 4\}}$

   (b) $B - A$ $\boxed{\{0\}}$

   (c) $A \times B$ $\boxed{\{(1, 0), (2, 0), (3, 0), (4, 0), (1, 1), (2, 1), (3, 1), (4, 1)\}}$

   (d) $P(B)$ $\boxed{\{\emptyset, \{0\}, \{1\}, \{0, 1\}\}}$

   (e) $B \times \mathbb{N}$ $\boxed{\{(0, 1), (0, 2), (0, 3), \ldots, (1, 1), (1, 2), (1, 3), \ldots\}}$