Sum/Product Notation

In induction proofs you'll frequently come across formulas involving sum ($\sum$) and product ($\prod$) notations. These notations are abbreviations for repeated sums or repeated products, and it is important to learn how to work with them, and get comfortable and secure in using these notation. The problems below are intended to practice these skills.

Some useful summation formulas: The following are formulas you should know. All are easy to prove by induction.

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=0}^{n} r^i = \frac{1-r^{n+1}}{1-r} \quad (r \neq 1), \quad \sum_{i=0}^{\infty} r^i = \frac{1}{1-r} \quad (|r| < 1).$$

Practice Problems

1. Write the given sums and products explicitly without using sum/product notations, (e.g., as $1+2+\cdots+n$ or $\underbrace{1+\cdots+1}_{n}$), then evaluate them. (Here n is a positive integer.)

   (a) $\sum_{i=1}^{n} 1$
   (b) $\prod_{i=1}^{n} i$
   (c) $\prod_{i=1}^{n} 2$
   (d) $\prod_{i=1}^{n} n^i$
   (e) $\sum_{i=1}^{n} n^i$
   (f) $\prod_{i=1}^{n} \frac{n+i}{i}$

2. Simplify, and evaluate if possible, the following sums by shifting the index or similar manipulations.

   (a) $\prod_{i=1}^{n}(i+2)$
   (b) $\sum_{i=0}^{n} a_{i+1}$
   (c) $\prod_{i=1}^{n}(n-i+1)$
   (d) $\sum_{i=0}^{n}(a_{i+1}-a_i)$
   (e) $\sum_{i=1}^{n}(4i-1)$
   (f) $\sum_{k=0}^{\infty} \frac{k+1}{k!}$ (Hint: Break the sum into two parts.)

3. Write the following in summation notation and evaluate using appropriate summation formulas.

   (a) $3+7+11+\cdots+(4n-1)$
   (b) $y^n + xy^{n-1} + x^2y^{n-2} + \cdots + x^{n-1}y + x^n$ $(x, y \neq 0)$.