Math 361 X1 **Spring 2003**

The Car/Goat Problem

The car/goat problem (also known as Monty Hall problem, after the name of the host of the TV quiz show where this problem originated) goes as follows. (This is the version published in Marylin vos Savant's column in *Parade Magazine* (see the problem labelled "Problem 1" in the article 'The Car-and-Goats Fiasco" handed out in class.)

One of three doors hides a car (all three equally likely) and the other two hide goats. You choose Door 1. The host, who knows where the car is, then opens one of the other two doors to reveal a goat, and asks whether you wish to switch your choice. Say he opens Door 3; should you stick with your original choice, Door 1, or switch to Door 2?

As stated, this problem is somewhat ambiguous. For example, it is not clear from the problem whether the host *deliberately* picked a door hiding a goat, or whether he picked one of Doors 2 and 3 at random and it just happened to hide a goat. With the following assumptions (which are rather natural assumptions, but not explicitly stated in the problem), the problem becomes precise.

(1) Each of the three doors is equally likely to hide the car; the other two doors hide a goat.

(2) You start by picking Door 1. You announce your choice, but the door remains closed.

(3) The host knows where the car is and always chooses a door that (i) does not hide the car and (ii) is different from the door you picked. Thus, since you picked door 1, his choices are limited to doors 2 and 3. If one of these two doors hides a car, he has no choice at all and is forced to pick the other door; if both doors hide a goat, he chooses one of doors 3 and 2 at random, with probability 1/2 each (e.g., by tossing a coin).

(4) The probabilities sought in the problem are *conditional* probabilities of winning *given that the host has opened Door 3*, under each of the following two strategies:

   (a) *The switching strategy:* You switch from door 1 to whichever door (among 2 and 3) the host has *not* opened.

   (b) *The no-switching strategy:* you stick with your original choice, i.e., door 1.

Problem: Set up this problem as a conditional probability problem, using the above assumptions, and compute the winning probabilities under strategies (a) and (b). Obviously, the strategy that results in the higher winning probability, is the one that you should choose.