Math 361 X1 **Spring 2003**

The Car/Goat Problem, II
Professors versus the General Public

The article "The Car-and-Goats Fiasco" handed out in class contained an interesting subproblem (subplot?) which concerned rates of disagreement (D) among respondents from the General Public (G) and those from Universities (U). Here is the relevant excerpt:

> [...] Nine-tenths of them [the respondents to Marylin's column] insisted that ... doors 1 and 2 were still equally likely [i.e., disagreed with Marylin's conclusion]. Of the respondents from the general public, 92% disagreed with her, while of the responses from universities, 65% disagreed. **It follows that 7.5% of the responses came from Universities. Therefore 5% were naysayers from universities [...]**

The statement in boldface is anything but obvious, but the point is that underlying mathematics is the same as that behind the car/goat problem. Here is how to set up and solve this problem:

Relevant events:

P = "respondent from general public"

U = "respondent from university"

D = "disagrees with Marylin"

Given data:

(1) $P(D) = 0.9$

(2) $P(D|P) = 0.92$

(3) $P(D|U) = 0.65$

To show: The two claims made in the above quote can be expressed as:

(4) $P(U) = 0.075$ ("7.5% of the responses came from Universities")

(5) $P(UD) = 0.05$ ("5% were naysayers from Universities"). (Note that the latter is to be interpreted as the probability of the intersection of D and U, not a conditional probability.)

Computations:

(4) First note that from the context of the problem it is clear that $P = U^c$, so $P(P) = 1 - P(U)$. Applying the total probability rule with the $B_i's$ being the sets P and U (which partition Ω since they are complements of each other), and with $A = D$, we get

$$P(D) = P(D|P)P(P) + P(D|U)P(U) = P(D|P)(1 - P(U)) + P(D|U)P(U).$$

The only unknown in this equation is $P(U)$, and substituting the given values for $P(D)$, $P(D|P)$ and $P(D|U)$, and solving for $P(U)$ gives the asserted value $P(U) = 0.075$.

(5) By the multiplication rule, $P(DU) = P(D|U)P(U) = 0.65 \cdot 0.075 \approx 0.05$.