Math 361X1, Spring 2003
Continuous distributions: definitions and properties

	Continuous	Discrete
Definition	X is continuous if $P(X = x) = 0$ for all x	X is discrete if X has finitely or countably infinitely many values
Distribution function (c.d.f.) Properties	$F(x) = P(X \leq x)$ $= \int_{-\infty}^{x} f(y)dy$ $F(-\infty) = 0$, $F(\infty) = 1$, F is non-decreasing	$F(x) = P(X \leq x)$
Density function Properties	$f(x) = F'(x)$ $f(x) \geq 0$ $\int_{-\infty}^{\infty} f(x)dx = 1$	$f(x) = P(X = x)$ $P(X = x) \geq 0$ $\sum_{\text{all } x} P(X = x) = 1$
Interval probabilities $P(a \leq X \leq b$	$\int_a^b f(x)dx (= F(b) - F(a))$	$\sum_{a \leq x \leq b} P(X = x)$
Expectation $E(X)$ Expectation of a function of X, $E(g(X))$	$\int_{-\infty}^{\infty} x f(x)dx$ $\int_{-\infty}^{\infty} g(x)f(x)dx$	$\sum_{\text{all } x} xP(X = x)$ $\sum_{\text{all } x} g(x)P(X = x)$
Variance Var(X)	$E(X^2) - E(X)^2$	same formula

Notes:

- For continous r.v.'s the density $f(x)$ cannot be computed directly; the formula $f(x) = P(X = x)$ is only valid when X is discrete. (In fact, the latter formula wouldn't make sense, since for continuous r.v.'s $P(X = x) = 0$ for all x.) Instead, to obtain the density one first needs to compute the corresponding c.d.f. via the definition $F(x) = P(X \leq x)$, then obtain $f(x)$ as the derivative $f(x) = F'(x)$.

- While the c.d.f. $F(x)$ plays an important role in continuous distributions (for the above reason), for discrete distributions it is largely irrelevant.

- For continuous r.v.'s, $<$ and $\leq$ are interchangeable, so that, e.g., $P(0 \leq X \leq 1) = P(0 < X < 1)$.

Important continuous distributions

- **Uniform**(a, b)**:**

  - **Density:** $f(x) = \frac{1}{b-a}$ $(a \leq x \leq b)$
  - **Mean:** $(a + b)/2$

    [no need to memorize the c.d.f. or variance]

- **Normal**(μ, σ^2)**:**

  - **Density:** $f(x) = \frac{1}{\sigma}\varphi\left(\frac{x-\mu}{\sigma}\right)$ $(-\infty < x < \infty)$ $(\varphi(z) = \frac{1}{\sqrt{2\pi}}e^{-z^2/2})$
  - **C.d.f.:** $F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$ $(-\infty < x < \infty)$
  - **Mean:** μ **Variance:** σ^2

- **Exponential**(λ)**:**

  - **Density:** $f(x) = \lambda e^{-\lambda x}$ $(0 < x < \infty)$
  - **C.d.f.:** $F(x) = 1 - e^{-\lambda x}$ $(0 < x < \infty)$
  - **Mean:** $1/\lambda$ **Variance:** $1/\lambda^2$