Math 361 X1 **Homework 4** **Due 2/21/2003**

Name (please print):

As usual, you have to solve the problems rigorously, using the methods introduced in class. An answer alone does not count. The problems in this assignment are intended as exercises in (i) modeling a given word problem within the success/failure framework (which requires, in particular, to say what a "trial" corresponds to and what "success" means, to specify the parameters n and p, and to express the events in question as success/failure events), and (ii) applying appropriate approximations to the binomial distribution (which, besides knowing the relevant formulas, requires deciding which approximation is appropriate). Therefore, you will not get credit if you attempt to do these problems by brute force, using the binomial distribution (which, in most cases, would be more difficult or impossible anyway).

Problem 1.

Find formulas for the following probabilities. You can leave answers in "unsimplified" form. However, in those cases where *approximate* formulas are requested (parts (b) and (d)), these formulas should be simple enough so that one could easily compute a numerical value with the aid of a basic, non-programmable calculator. Thus, for example, a summmation involving 100 terms would not be acceptable, nor would formulas involving large factorials (such as 100! or binomial coefficients).
(a) An **exact** formula for probability that in a class of 365 students at least two have their birthday on January 1. (You may assume that there are 365 possible birthdays.)
(b) An **approximate** formula for the probability computed in (a).
(c) An **exact** formula for the probability that in a country with a population of 365,000,000 people exactly 1,000,000 have their birthday on January 1.
(d) An **approximate** formula for the probability computed in (c).

Problem 2.

[2.2:6] To estimate the percent of district voters who oppose a certain ballot measure, a survey organization takes a random sample of 200 voters from a district. If 45 % of the voters in the district oppose the measure, estimate the chance that (a) exactly 90 voters in the sample oppose the measure; (b) more than half the voters in the sample oppose the measure.

Problem 3.

[2.2:10] A probability class has 30 students. As part of an assignment, each student tosses a coin 200 times and records the number of heads. Approximately what is the chance that no student gets exactly 100 heads?

Problem 4.

[2.2:14] A company has developed electronic devices which work with probability 0.95, independently of each other. The devices are shipped in boxes containing 400 each. (a) What percentage of the boxes contains at least 390 working devices? (b) The company wants to guarantee that at least k devices in each box work. What is the largest value of k such that 95 % of the boxes meet this warranty?

Problem 5.

[2.4:6(a)] A box contains 1000 balls, of which 2 are black and the rest are white. Which of the following is most likely to occur in 1000 draws with replacement from the box? (i) Fewer than 2 black balls; (ii) exactly 2 black balls; (iii) more than two black balls. Do this problem by computing numerically, **using appropriate approximations**, the three probabilities involved.

Problem 6.

The mathematics department of a large state university has funds for 71 assistantships for new graduate students. Past experience has shown that only 50 % of those applicants who are offered an assistantship accept the offer.
(a) Suppose the department makes exactly 100 offers. Determine a value x such that, with (approximately) 95.44% probability, the number of those accepting the offer is between $50 - x$ and $50 + x$.
(a) How many offers (approximately) can the Department make and still be 97.72 % certain that no more than 71 offers get accepted?