Math 361 X1 **Homework 6** **Due 3/17/2003**

Name (please print):

Extended deadline. Because of the Engineering Open House on Friday, 3/14, the deadline for this assignment has been extended to the following Monday (3/17).

Instructions. As usual, answers alone, like the ones you find in the back of the book, will not earn you any credit—you are expected to show all work and justify (rigorously) your answers. Thus, for example, if a problem asks to find a distribution, just writing down the distribution table will not earn you credit; you need to show how you got the entries of the table. A distribution table doesn't necessarily have to be written down in the form of a table (which would be rather impractical, for example, in the case of a 6 by 6 joint distribution table with 36 entries). You can specify the table by giving a formula for a general entry $P(i)$ or $P(i,j)$, along with ranges for the values i and j. (Be sure to include the range – i.e., the list of possible values – along with each distribution.) Problems asking for the distribution of a random variable almost always boil down to traditional probability computations (i.e., of the type that came up in Chapters 1 and 2), which may be very simple in some cases, but also (in one or two problems) can be on the difficult side. These probability computations should be done following the methods of Chapters 1 and 2, such as equally likely probability models, success/failure models, "assembly level" probability computations in S/F models, etc.; if necessary, review of those methods.

Problem 1.

[3.1:4] Let X_1 and X_2 be the numbers obtained on two rolls of a fair die. Let $Y_1 = \max(X_1, X_2)$ and $Y_2 = \min(X_1, X_2)$. Find the joint distribution of (a) (X_1, X_2), (b) (Y_1, Y_2).

Problem 2.

[3.1:6] A fair coin is tossed three times. Let X be the number of heads on the first two tosses, Y the number of heads on the last two tosses. (a) Find the joint distribution of X and Y. (b) Are X and Y independent? Justify your answer rigorously. (c) Find the distribution of $X + Y$.

Problem 3.

[3.1:9] A box contains 8 tickets. Two are marked 1, two marked 2, two marked 3, and two marked 4. Tickets are drawn at random from the box without replacement until a number appears that has appeared before. Let X be the number of draws that are made. Find the distribution of X.

Problem 4.

[3.1:13(a), variant] A box contains $2n$ balls of n different colors, with 2 of each color. Balls are picked at random from the box with replacement until two balls of the same color have appeared. Let X be the number of draws made. (a) Find a formula for $P(X > k)$; (b) (not in book) Using the result of part (a), determine the distribution of X.

Problem 5.

[3.1:14(a),(b),(e)] **Baseball World Series, part II.** In the World Series, teams A and B play until one team has won four games. (There are no ties.) Assume that each game played is won by team A with probability p, independently of all other games. (a) For $g = 4, 5, 6, 7$ find a formula in terms of p and $q = 1 - p$ that team A wins in g games. (b) What is the probability that team A wins the World Series, in terms of p and q ? (e) Let G represent the number of games played. What is the distribution of G?

Problem 6.

[3.1:15(a),(d),(f)] Let X and Y be independent, each uniformly distributed on $\{1, 2, \ldots, n\}$. Find (i) $P(X = Y)$, (ii) $P(\max(X, Y) = k)$ for $k = 1, 2, \ldots, n$, (iii) $P(X + Y = k)$ for $k = 2, 3, \ldots, 2n$.