Math 361 X1 **Homework 7** **Due 3/14/2003**

Name (please print):

As always, you need to show work. Problems asking for a distribution of a r.v. require you to (a) list all values of the r.v., and (b) compute, for each of these values, the probability with which the value is taken on. Part (b) is where the real work goes, and you need to show how you did these probability computations; just writing down a distribution table will not earn you any point. The solutions to HW 6 give you an idea of what is expected.

Problem 1.

[3.2:8] Suppose $E(X^2) = 3$, $E(Y^2) = 4$, $E(XY) = 5$. Find $E[(X+Y)^2]$.

Problem 2.

Let X and Y be the two numbers appearing on two independent rolls of a die. Find (a) $P(|X-Y| \leq 1)$; (b) $E(2^X)$; (c) $E(2^{X+Y})$.

Problem 3.

[3.2:13,variant] Suppose a fair die is rolled 10 times. Let $X_1, X_2, \ldots, X_{10}$ denote the 10 numbers obtained, let $S = \sum_{i=1}^{10} X_i$ be the sum of these numbers and $X^* = \max(X_1, \ldots, X_{10})$ the largest of these numbers. (a) Find the expectation of S. (b) Find the distribution of X^*. (c) Find the expectation of X^*.

Problem 4.

[3.R:7] **Airline overbooking, III.** Suppose an airline accepted 12 reservations for a commuter plane with 10 seats. They know that 7 reservations went to regular commuters who will show up for sure. The other 5 passengers will show up with a 50 % chance, independently of each other. (a) Find the probability that the flight will be overbooked; (b) Find the probability that there will be empty seats; (c) Let X be the number of passengers turned away (in case the flight is overbooked; otherwise, $X = 0$). Find $E(X)$.

Problem 5.

[3.R:8, variant] A box contains 2 red and 4 black balls. Balls are drawn from the box until 2 black balls have been obtained. Let X denote the number of draws that have to be made. Find the distribution of X if (i) the draws are made without replacement, (ii) the draws are made with replacement.

Problem 6.

[3.3:8] Let A_1, A_2, and A_3 be events with probabilities 1/5, 1/4, and 1/3, respectively. Let N be the number of events that occur.
(a) Write down a formula for N in terms of indicators.
(b) Using the formula of part (a), find $E(N)$.

Problem 7.

Records: A die is rolled repeatedly. Saying that a record occurs at roll n means that the number appearing on the nth roll is strictly greater than each of the numbers that have appeared on previous rolls. The number appearing on the first roll is, by default, considered to be a record.
(a) Find the probability that a record occurs at the 4th roll.
(b) Find a general formula (which may involve a summation) for the probability that a record occurs at the nth roll, for $n = 2, 3, 4, \ldots$.
(c) Find the expected number of records in the first 4 rolls.

Problem 8 (extracredit)

Here is a problem that has an unexpected (and hard to guess) answer, which is not easy to derive theoretically, but which can relatively easily be obtained with a computer simulation:
Suppose you pick random numbers $t_1, t_2, t_3, \ldots$ from the interval $[0, 1]$ and add these up until the sum exceeds 1. Let N denote the number of random numbers you have to pick to get a sum larger than 1, i.e., N is the smallest index n such that $t_1 + t_2 + \cdots + t_n > 1$. Thus, N is a random variable with values $2, 3, 4, \ldots$, and it makes sense to ask for the mean, or average value, $E(N)$ of N. Use a computer simulation to determine an approximate numerical value for $E(N)$, then try to guess from that numerical value what the exact value is.