# Math 361 X1, Spring 2003

## Normal approximation, part II

**Definition of normal distribution:**

$$\Phi(x) = \int_{-\infty}^{x} \phi(t)dy, \quad \text{where } \phi(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}.$$

**Standardization:** Given any $X$, set $X^* = \frac{X-\mu}{\sigma}$, where $\mu = E(X)$, $\sigma = SD(X)$. Then $X^*$ has mean 0 and standard deviation 1.

**Normal($\mu$, $\sigma^2$) distribution:** $X$ has normal($\mu$,$\sigma^2$) distribution if

$$P(X \leq x) = \Phi\left(\frac{x-\mu}{\sigma}\right), \quad \text{or equvialently } P(X^* \leq x) = \Phi(x)$$

**Normal approximation to the binomial distribution:** If $X$ is binomial($n$,$p$) distributed, then (under certain conditions)

$$P(X \leq x) \approx \Phi\left(\frac{x-np}{\sqrt{np(1-p)}}\right), \quad \text{or equvialently } P(X^* \leq x) \approx \Phi(x)$$

**Normal approximation to sums of i.i.d. (independent identically distributed) random variables ("central limit theorem"):** Suppose $X_1, X_2, \ldots$ are i.i.d., with mean $\mu = E(X_1)$ and standard deviation $\sigma = SD(X_1)$, and let $S_n = X_1 + X_2 + \cdots + X_n$. Then, for large $n$,

$$P(S_n \leq x) \approx \Phi\left(\frac{x-n\mu}{\sigma\sqrt{n}}\right), \quad \text{or equvialently } P(S_n^* \leq x) \approx \Phi(x)$$

**Remarks:**

1. The formulas involving the standardized quantities, $X^*$ and $S_n^*$, look simpler and more elegant, but for applications the first formulas given (without the standardization) are more useful.

2. Formulas for $P(a \leq X \leq b)$ or $P(a \leq S_n \leq b)$ can be obtained by subtracting the corresponding $\Phi$-terms with $x = a$ and $x = b$.

3. In the above formulas $\leq$ and $<$ are interchangeable; the error made by replacing $\leq$ by $<$, for example, is absorbed by the error that is present anyway due to the approximate nature of the formulas.

4. Do not mix up the formulas for the normal approximation to the binomial distribution with those for the approximation to sums of i.i.d. r.v.; the $\mu$ and $\sigma$ in the latter formulas correspond to $p$ and $\sqrt{p(1-p)}$, not to $np$ or $\sqrt{np(1-p)}$.