The Chinese Remainder Theorem

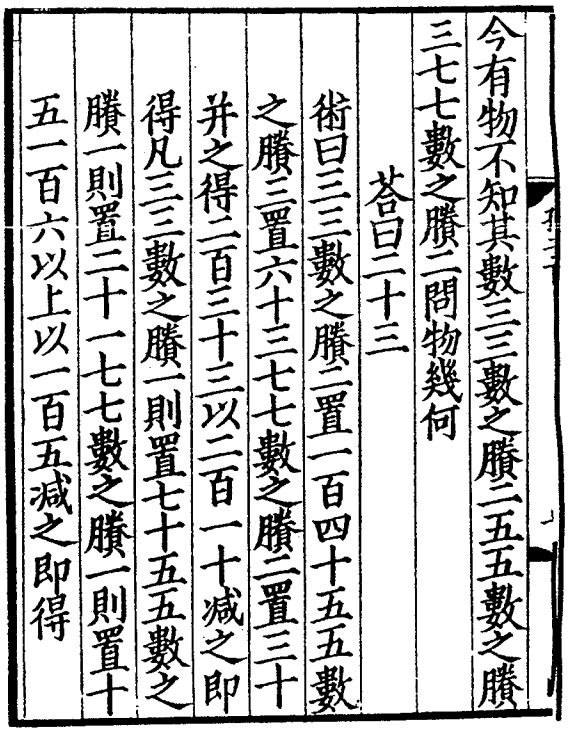
今有物不知其數三三數之賸二五五數之賸
三七七數之賸二問物幾何
答曰二十三
術曰三三數之賸二置一百四十五五數
之賸三置六十三七七數之賸二置三十
并之得二百三十三以二百一十減之即
得凡三三數之賸一則置七十五五數之
賸一則置二十一七七數之賸一則置十
五一百六以上以一百五減之即得

Figure 49. The famous Sun Tzŭ problem, the oldest instance of the remainder theorem. From the *Sun Tzŭ suan-ching* 孫子算經 (*T'ien-lu Lin-lang ts'ung-shu* 天祿琳琅叢書 ed.), *ts'e* 15, C, p. 10b.

The literal translation is as follows:

"We have things of which we do not know the number; if we count them by threes, the remainder is 2; if we count them by fives the remainder is 3; if we count them by sevens the remainder is 2. How many things are there? Answer: 23.[8] Method: if you count by threes and have the remainder 2, put 140.

If you count by fives and have the remainder 3, put 63.

If you count by sevens and have the remainder 2, put 30.

Add these [numbers] and you get 233.

From this subtract 210 and you have the result.

For each unity as remainder when counting by threes, put 70.

For each unity as remainder when counting by fives, put 21.

For each unity as remainder when counting by sevens, put 15.

If [the sum] is 106 or more, subtract 105 from this and you get the result."[9]