

Name:

Collaborator(s)¹:

Math 453, Section X13, Prof. Hildebrand, Spring 2011

HW Assignment 5, due Monday, 3/7/2011

Instructions

- **Rules:** The usual: Write your name on the cover sheet and staple the sheet to the assignment. Do the problems in order, and make sure that each problem is clearly labelled. The assignment is due in class at the above due date.

Solutions, rather than answers, are expected for all problems. Write legibly, using proper mathematical notation and terminology, and in complete sentences. For proofs you can use any result covered in class, the class handouts, and the relevant sections of the Strayer text.

- **Hints for this assignment:** Several problems (such as 13 and 14) involve finding all n -values for which $\phi(n)$ has a particular form. Follow the systematic approach illustrated in class for problems of this type: First split off the odd part of n , i.e., write $n = 2^\alpha m$, where m is odd and α is a nonnegative integer, and then (if necessary) consider the prime factorization of m .
- **Additional suggested practice problems (not to be turned in):** Chapter 2: 68(a)(b), 69(a)(b)(c)(d). Chapter 3: 5 (non-graded parts), 10(a)(b)(c), 13(a), 30(a)(b)(c), 31(a)(c)(d), 42(a)(b)(c). These problems are mostly routine computations, or are variations of a problem from the list below.

HW 5 Problems (Chapter 3 of Strayer)

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|----------------------|-----------|-------------------------------------|
| 1. *3 (Hint in book) | 5. *13(d) | 9. *32 |
| 2. *5(d)(f)(h) | 6. *14(b) | 10. *42(h) |
| 3. *7(a)(b) | 7. *30(h) | 11. *47 (Hint: Clear denominators!) |
| 4. *10(h) | 8. *31(e) | |

12. Extra credit problems.²

- Find *all* solutions $n \in \mathbb{N}$ to the equation $\phi(n) = 24$. (The point of this problem is to discover all solutions via a *systematic* paper/pencil approach that guarantees that one has found all solutions. Using brute force computation, or tables of the Phi function, would defeat that purpose, and won't earn credit.) (Fun fact: This problem appears as Exercise 3 in Chapter II of Carmichael's *The Theory of Numbers*, just a few problems ahead of the (in)famous exercise that is now known as Carmichael's Conjecture. In contrast to the latter, this one is very much doable!)
- Find all solutions $n \in \mathbb{N}$ to the equation $\phi(n) = 2^{10}$ (again via a systematic approach—see the above comment). For an extra challenge (and possible additional bonus points), see if you can determine the *number* of positive integers n for which $\phi(n) = 2^{10000000}$.
- HW 4 EC Problem (about a self-correcting ISBN numbering scheme). Since there were no takers on this problem, the problem remains on the table, just like the jackpot in a lottery ... See HW 4 for the exact statement.

¹If you worked with another student or in a small group on this assignment, list the names of all students involved.

²**No hints, help, or group work on extra credit problems, as this would defy the intent of these problems.** The problems require nothing more than the techniques and results covered class, but are out of the ordinary in one way or another, and require more thought and insight than the typical regular homework problem. If such a problem piques your interest, give it a try; otherwise, no harm done—problems at this level aren't exam material.