

Name:

Collaborator(s)¹:

Math 453, Section X13, Prof. Hildebrand, Spring 2011

HW Assignment 6, due Monday, 3/14/2011

Instructions

- **Rules:** The usual: Write your name on the cover sheet and staple the sheet to the assignment. Do the problems in order, and make sure that each problem is clearly labelled. The assignment is due in class at the above due date.

Solutions, rather than answers, are expected for all problems. Write legibly, using proper mathematical notation and terminology, and in complete sentences. For proofs you can use any result covered in class, the class handouts, and the relevant sections of the Strayer text.

- **Hints for this assignment:** Many of the problems in this assignment (e.g., 40, 63, 64, 67) amount to proving identities between arithmetic functions, i.e., proving (*) $F(n) = G(n)$ for all $n \in \mathbb{N}$, where F and G are arithmetic functions. **If** both sides are multiplicative (as is often the case, though in some cases this requires some justification, e.g., by citing appropriate theorems), then it suffices to show that (*) holds for prime powers, i.e., that (**) $F(p^a) = G(p^a)$ for any prime power p^a . At prime powers, arithmetic functions can (usually) be computed directly from the definition, so proving (**) is usually quite routine.

(A problem of this type already occurred in HW 5 (Problem 7, Chapter 3).)

HW 6 Problems (Chapter 3 of Strayer)

1. *40
2. *59(a)(b)(c)
3. *63
4. *64
5. *65 (see hint in book)
6. *67(a)(b)(c)(d)

7. Extra credit problems.²

- (i) **Dirichlet squareroot of 1:** Given an arithmetic function f , a “Dirichlet squareroot” of f is a function g such that $g * g = f$. Prove that the function $\mathbf{1}$ has a unique multiplicative Dirichlet squareroot g , try to find an explicit formula for the values of g at prime powers, and/or use a computer program to compute $g(n)$ explicitly for the first few hundred values—far enough to be able to answer questions like the following: What is $g(453)$, or $g(2011)$, or—for brave souls— $g(2^{2011})$? (This problem is somewhat open-ended and has subquestions of both theoretical and computational flavor.)
- (ii) **Dirichlet powers of 1:** In the opposite direction, one can define the k -th “Dirichlet power” of $\mathbf{1}$ as the function $f_k = \underbrace{\mathbf{1} * \cdots * \mathbf{1}}_k$. In the case $k = 2$, this just gives the divisor function $\nu(n)$; i.e., $f_2(n)$ counts the number of (positive) divisors of n . Try to describe, in a similar *simple* way, what $f_k(n)$ counts. Also, find, if possible, formulas for $f_k(2^a)$, for general k and a .

¹If you worked with another student or in a small group on this assignment, list the names of all students involved.

²No hints, help, or group work on extra credit problems, as this would defy the intent of these problems. The problems require nothing more than the techniques and results covered class, but are out of the ordinary in one way or another, and require more thought and insight than the typical regular homework problem. If such a problem piques your interest, give it a try; otherwise, no harm done—problems at this level aren’t exam material.