

Name:

Collaborator(s)¹:

Math 453, Section X13, Prof. Hildebrand, Spring 2011

HW Assignment 7, due Friday, 4/1/2011 (NOTE DUE DATE)

Instructions

- **Rules:** The usual: Write your name on the cover sheet and staple the sheet to the assignment. Do the problems in order, and make sure that each problem is clearly labelled. The assignment is due in class at the above due date.
- **Hints for this assignment:**
 - **Problem 6:** We know that there are exactly $(p - 1)/2$ quadratic residues. To show that a given list of $(p - 1)/2$ numbers is equal, modulo p , to the list of quadratic residues, it suffices to show (why?) that (a) each of the numbers in this list is congruent to a quadratic residue, and (b) no two numbers on the list can be congruent to the same quadratic residue. (This is a general principle that has come up before, e.g., in connection with complete or reduced residue systems.)
 - **Problem 7(a):** The hint in the book says to use the formula for the sum of the first n squares, $1^2 + 2^2 + \cdots + n^2 = n(n + 1)(2n + 1)/6$. It only remains to connect such a sum of squares with a sum of quadratic residues ...
 - **Problems 16, 17, 18:** These problems (and others such as 18) illustrate the usefulness of the congruence and/or Legendre symbol notations. In the given form, the questions may seem intractable, but when translated into *appropriate* language/notation, the solution becomes a simple application of known results in that language.
 - **Problem 20:** Hints: (i) Keep in mind that a Legendre symbol is either $+1$ and -1 . (ii) When is a sum of $+1$'s and -1 's equal to 0? (iii) You'll need two theorems, one from 4.1 and another from 4.2.
- **Additional suggested practice problems (not to be turned in):** Chapter 4: 1(a)(c), 2, 3, 10(a)(c), 14(a)(c) These problems are mostly routine computations, or are variations of a problem from the list below. For additional examples and practice material, see the texts on course reserve for Math 453 in the Math Library. The closest in content to the Strayer text is Rosen's "Elementary Number Theory", a much heftier text with numerous examples and problems covering all levels.

HW 7 Problems (Chapter 4 of Strayer)

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|-------------|-----------|---------------|
| 1. *5(a)(b) | 5. *12(b) | 9. *20 |
| 2. *6 | 6. *16(a) | 10. *24(a)(b) |
| 3. *7(a) | 7. *17(a) | |
| 4. *10(b) | 8. *18 | |

11. Extra credit problems.²

The two EC problems from HW 6 (Dirichlet squareroot of 1 and Dirichlet powers of 1—see HW 6 for precise statements) were unclaimed, so they remain on the table for one more round. If you want a (modest) challenge (and a chance to earn extra credit!), give it a try over the break!

¹If you worked with another student or in a small group on this assignment, list the names of all students involved.

²No hints, help, or group work on extra credit problems, as this would defy the intent of these problems.

The problems require nothing more than the techniques and results covered class, but are out of the ordinary in one way or another, and require more thought and insight than the typical regular homework problem. If such a problem piques your interest, give it a try; otherwise, no harm done—problems at this level aren't exam material.