

Math 453, Section X, Spring 2008
HW Assignment 7, due Monday, 3/31/2008

Name (print please):

- **Rules:** The usual: Write your name on the cover sheet and staple the sheet to the assignment. Only the problems marked by an asterisk are to be turned in. Do these problems in order, and make sure that each problem is clearly labelled. Open House Hours are, as usual, Wednesdays and Thursdays, at 5 pm, in 141 Altgeld. (Thursday is better since on Wednesday my other class has priority.)

Comments and Hints

- **Chapter 3, 59(b):** This boils down to a calculation of σ values, using the properties and formulas of the σ -function, and the definition of a Mersenne prime. (Part (a) is just a special case of (b); you might want to do it in this case first.)
- **Problem 6:** We know that there are exactly $(p - 1)/2$ quadratic residues. To show that a given list of $(p - 1)/2$ numbers is equal, modulo p , to the list of quadratic residues, it suffices to show (why?) that (a) each of the numbers in this list is congruent to a quadratic residue, and (b) no two numbers on the list can be congruent to the same quadratic residue. (This is a general principle that has come up before, e.g., in connection with complete or reduced residue systems.)
- **Problem 7(a):** The hint in the book says to use the formula for the sum of the first n squares, $1^2 + 2^2 + \dots + n^2 = n(n + 1)(2n + 1)/6$. It only remains to connect such a sum of squares with a sum of quadratic residues ...
- **Problems 16, 17:** These problems (and others such as 18) illustrate the usefulness of the congruence and/or Legendre symbol notations. In the given form, the questions may seem intractable, but when translated into *appropriate* language/notation, the solution becomes a simple application of known results in that language.
- **Problem 20:** Hints: (i) Keep in mind that a Legendre symbol is either $+1$ and -1 . (ii) When is a sum of $+1$'s and -1 's equal to 0? (ii) You'll likely need two theorems, one from 4.1 and another from 4.2.

HW 7 Problems (all except the first are from Chapter 4 of Strayer)

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| 1. * Ch. 3, 59(b)) | 8. * 17(a) |
| The remaining problems in this set are all from Chapter 4. | 9. * 20 |
| 2. * 6 | 10. 1(a)(c) |
| 3. * 7(a) | 11. 2 |
| 4. * 10(b) | 12. 10(a)(c) |
| 5. * 12(b) | 13. 14(a)(c) |
| 6. * 13(b) | |
| 7. * 16(a) | |