Some important discrete distributions

1. **Binomial:**
   - **Parameters:** n (positive integer), p ($0 \le p \le 1$)
   - **P.m.f.:** $p(k) = \binom{n}{k} p^k (1-p)^{n-k} \quad (k = 0, 1, 2, \dots, n)$
   - **Expectation and variance:** $\mu = np$, $\sigma^2 = np(1-p)$
   - **Arises as:** Distribution of *number* of successes in success/failure trials ("Bernoulli trials").
2. **Poisson:**
   - **Parameter:** $\lambda > 0$
   - **P.m.f.:** $p(k) = e^{-\lambda} \frac{\lambda^k}{k!} \quad (k = 0, 1, 2, \dots)$
   - **Expectation and variance:** $\mu = \lambda$, $\sigma^2 = \lambda$
   - **Exponential series formula:** $\sum_{n=0}^{\infty} \frac{\lambda^n}{n!} = e^{\lambda}$
   - **Arises as:** Distribution of number of occurrences of *rare* events, e.g., accidents, disasters.
   - **Poisson approximation to binomial distribution:** A binomial distribution in which n is large and p correspondingly small (so that np is of moderate size) is approximately Poisson with parameter $\lambda = np$; i.e., if X has binomial distribution with parameters n and p as above, then $P(X = k) \approx e^{-\lambda} \lambda^k / k!$, where $\lambda = np$.
3. **Geometric:**
   - **Parameter:** p ($0 < p < 1$)
   - **P.m.f.:** $p(k) = (1-p)^{k-1} p$ ($k = 1, 2, \dots$)
   - **Expectation:** $\mu = 1/p$.
   - **Geometric series formula:** $\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$ ($|r| < 1$)
   - **Arises as:** Distribution of *time (trial) of first success* in a success/failure trial sequence.
4. **Negative binomial:**
   - **P.m.f.:** $p(k) = \binom{k-1}{r-1} (1-p)^{k-r} p^r, \quad k = r, r+1, \dots$
   - **Expectation:** $\mu = r/p$. (This is exactly r times the expectation of a geometric distribution. In other words, the expected time of the r-th success is r times the expected time of the 1-st success.)
   - **Arises as:** Distribution of *time (trial) of r-th success* in asuccess/failure trial sequence. Generalizes the geometric distribution, which corresponds to the case $r = 1$.
5. **Hypergeometric:**
   - **Parameters:** N, m, n
   - **P.m.f.:** $p(k) = \binom{m}{k} \binom{N-m}{n-k} / \binom{N}{n}$ ($k = 0, 1, \dots, n$)
   - **Expectation:** $\mu = (m/N)n$. (In other words, the expected number of red balls in the sample is exactly the "fair share", namely the proportion of red balls in the entire box (i.e., m/N) times the number of balls in the sample (i.e., n).)
   - **Arises as:** The distribution of the number of red balls, if n balls are drawn without replacement from a box containing a total of N balls, of which m are red.