Course Information Sheet
Math 531: Analytic theory of numbers
Fall 2005

Time and location: MWF 11:00 – 11:50, 443 Altgeld Hall

Instructor: A.J. Hildebrand, 241 Illini Hall, phone 244-7721, email `hildebr@math.uiuc.edu`

Course description: Analytic number theory deals with problems and results in number theory that can be approached with tools of analysis. The most famous result of this type is the prime number theorem, which states that the number of primes below x is asymptotically equal to $x/\log x$. This is a result that the great number theorists of the 19th century and before believed to be true, but were unable to prove. A proof was finally given at the end of the 19th century by Hadamard and de la Vallée Poussin, using a new analytic approach, based on ideas of Bernhard Riemann.

A proof of the prime number theorem will form the core and highlight of this course. Other topics include the behavior of so-called arithmetic functions, and problems of a probabilistic flavor, such as the following: What is the probability that two randomly chosen integers are relatively prime? (Answer: $6/\pi^2$.) What is the average number of divisors of an integer of size x? (Answer: about $\log x$.) What is the average number of *prime* divisors of an integer of size x? (Answer: about $\log\log x$.)

Prerequisites: The official prerequisites for this course are Math 448 (Complex Analysis) and either Math 417 (Abstract Algebra) or Math 453 (Elementary Number Theory). However, more important than any technical knowledge is the "mathematical maturity" acquired through courses at this level. In particular, students must feel at ease with $\epsilon-\delta$ arguments. Results from elementary number theory will be developed as needed during the course, so prior knowledge of elementary number theory is not necessary.

Text: I will distribute my own lecture notes, so there is no need to purchase a book. The following books can serve as additional references and will be placed on reserve in the library.

- T. Apostol, "Introduction to Analytic Number Theory"
- P.T. Bateman and H.G. Diamond, "Analytic Number Theory"
- K. Chandrasekharan, "Introduction to Analytic Number Theory"
- H. Davenport, "Multiplicative Number Theory"

Among these, the books by Apostol and Bateman/Diamond are closest in spirit and content to this course. Davenport's book is a classic (it first came out some 50 years

ago), but is more challenging than the other texts. Chandrasekharan's book covers elementary number theory very well and would be a good place to review material from elementary number theory, but its coverage of analytic number theory is not nearly as deep or broad as our course.

Detailed Syllabus:

0. Review of elementary number theory (divisibility, gcd, lcm, congruences, primes, fundamental theorem of arithmetic)
1. Arithmetic functions I: elementary theory (multiplicative and additive functions, Dirichlet multiplication, Moebius inversion)
2. Arithmetic functions II: averages and order of magnitude estimates.
3. Distribution of primes I: elementary theory (statements equivalent to the prime number theorem, Chebyshev and Mertens estimates)
4. Arithmetic functions III: Dirichlet series and Euler products
5. Distribution of primes II: Analytic proof of the prime number theorem
6. Dirichlet's theorem on primes in arithmetic progressions

The above material forms the core of the course, and the syllabus for the Math 531 Comprehensive Exam. Additional topics may be covered, depending on the amount of time left over and audience preferences.

Grading: The course grade will be based on homework assignments, two midterm exams, and a final (scheduled for December 15, 8 - 11 am), with each component counting roughly 1/3 towards the course grade, though I may make small adjustments to this policy, e.g., by providing opportunities to earn extra credit through special projects or seminar presentations.

Course web page: `http://www.math.uiuc.edu/~hildebr/531/`. I will use this page to post there lecture notes, hw assignments and solutions, and general anouncements.