

Asymptotic Analysis  
Problem Set 5  
Due April 26, 2005

No group work on the final two problems!

**Problem 26**

Obtain an estimate for  $\int_0^\pi x^n \sin x dx$  to within an error of  $1 + O(1/n)$  (or better) as  $n$  tends to  $\infty$ .

**Problem 27**

Derive an asymptotic estimate for  $\int_0^1 e^x x^n (1+x^2)^{-n} dx$  with a relative error of  $O(1/\sqrt{n})$  (or better) as  $n \rightarrow \infty$ .

**Problem 28**

For  $\alpha > 1$  let  $I(t, \alpha) = \int_0^t \cos(x^\alpha) dx$ . Show that the integral  $I(t, \alpha)$  converges as  $t \rightarrow \infty$ , and obtain an estimate, as good as you can, for  $I(t, \alpha)$ . (It is very easy to get an estimate with error  $O(t^{1-\alpha})$ . With more work one can improve this to  $O_\alpha(t^{1-2\alpha})$ .)

**Problem 29**

Obtain an asymptotic estimate (as  $t \rightarrow \infty$ ) for the integral  $I(t) = \int_0^\infty x^{-x} e^{xt} dx$ .

**Problem 30**

Let  $\lambda > 1$  be fixed and let

$$I_\lambda(n) = \int_0^\infty \left(1 + \frac{x}{n}\right)^n e^{-\lambda x} dx.$$

Obtain an estimate for  $I_\lambda(n)$ , as  $n \rightarrow \infty$ , with error term  $O_\lambda(1/n)$ .