

## UIUC Mock Putnam Exam 1/2002

**Problem 1.** Show that  $\log_{2002} 2003 + \log_{2003} 2002 > 2$ .

**Problem 2.** Show that the number

$$\frac{2^3}{3^2} + \frac{4^3}{5^2} + \frac{6^3}{7^2} + \cdots + \frac{2002^3}{2003^2}$$

is not an integer.

**Problem 3.** Find the sum of the infinite series

$$\sum_{n=3}^{\infty} \frac{1}{\binom{n}{3}}.$$

**Problem 4.** Find all positive integers  $n$  which are equal to the sum of digits of  $n^2$ .

**Problem 5.** Is there a prime number  $p$  and a sequence  $(P_n)_{n \in \mathbf{N}}$  of integer points (i.e., points with integer coordinates) in the plane such that every integer point in the plane belongs to the sequence  $(P_n)_{n \in \mathbf{N}}$  and the distance between any two consecutive points  $P_n$  and  $P_{n+1}$  equals  $p$ ?