

UIUC Mock Putnam Exam 4/2002

Problem 1. Without numerical calculations, determine which of the two numbers $2.01^{1.99}$ and $1.99^{2.01}$ is larger.

Problem 2. Let $f(x) = x^3 e^{x^2} (1 - x^2)^{-2}$. Find $f^{(2002)}(0)$. (Here, $f^{(n)}$ denotes the n th derivative of f .)

Problem 3. Find a formula for

$$(1^2 + 1)1! + (2^2 + 1)2! + \cdots + (n^2 + 1)n!$$

Problem 4. Show that $\int_0^1 x^{-x} = \sum_{n=1}^{\infty} n^{-n}$.

Problem 5. Evaluate the integral

$$\int_0^{\pi/2} \frac{dx}{1 + (\tan x)^{\sqrt{2}}}.$$