

UIUC Mock Putnam Exam 2/2003
Advanced Version

Problem 1. Evaluate the integral

$$I = \int_0^{\pi} \ln(\sin x) dx.$$

Problem 2. Let a, b, c denote distinct integers. Does there exist a polynomial with integral coefficients that cyclically permutes these values in the sense that $P(a) = b$, $P(b) = c$ and $P(c) = a$? Explain!

Problem 3. [Putnam 1966, A3] Let $0 < x_0 < 1$, and $x_{n+1} = x_n(1 - x_n)$ for $n \geq 0$. Prove that the limit $\lim_{n \rightarrow \infty} nx_n$ exists and is equal to 1.

Problem 4. Evaluate $\sum_{k=0}^n \binom{n}{k}^2 (-1)^k$.